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Tuning of a PI Controller to Control Second-order-like Processes

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Abstract:

This paper investigates the tuning of a PI controller for use with second-order-like processes having natural frequency between 1 and 9 rad/s and damping ratio between 0.2 and 2. The controller is tuned using MATLAB optimization toolbox and an ITAE performance index. The tuning results are presented in five tables for five natural frequencies and 10 damping ratios. A case study is suggested to investigate the effectiveness of the proposed tuning technique where another tuning technique for the PI controller is compared with the present tuning technique for both reference and disturbance inputs.

Keywords — **Second-order-like processes, PI controller tuning, minimum ITAE standard forms, control system performance.**

I. INTRODUCTION

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Many processes and applications can be assumed as second-order-like ones. On the other hand the proportional integral (PI) controller still in use to control too many processes. It is one of the PID controller family which are the first generation of PID controllers. Lot of efforts were paid to tune the PI controller when used with specific processes. Here are some of the efforts regarding this objective:

Dwyer (2000) provided tuning rules for the PI control of SISO processes with time delay. He presented tuning rules for Ziegler-Nichols (1942), Astrom-Hagglund (1995), Chien et al. (1952), Murril (1967), St.Clair (1988), Zhuang-Atherton (1993), Rovira et al. (1969), Haalman (1965), Remberton (1972), Smith-Corripio (1985), Schneider (1988), Hang et al. (1999)Voda-Landeu (1995) and Lee et al. (1998). He presented the gain and phase margins of the control system against the ratio of time delay to time constant of the process [1]. Khan and Gorez (2003) showed that the design rules proposed by them were appropriate for self-regulating processes with normalized dead time less than 0.8. They presented the tuned parameters of the PI controller in a tabular form for a specific process with different gain and time constant using Ziegler-Nichols, Cohen-Coon, Balanced tuning and Astrom-Hagglund tuning methods [2]. Foley, Ramharack and Copeland (2005) compared the Skogestad internal model control, direct synthesis for disturbance rejection and Wang-Shao tuning algorithms with the IMC improved PI technique on first-order plus dead time processes. They provided recommendations for selecting the most appropriate tuning technique for a given application [3].

Wahynggoro and Saad (2008) discussed the modelling and simulation of DC servomotor control using MATLAB/Simulink using a fuzzy-scheduled PID and a fuzzy-logic based self-tuning of a PI controller. They handled two control modes for speed and position control. They showed that the PI controller offered better performance compared to the PID control for the speed control of the DC servomotor [4]. Haugen (2010) presented a number of PI controller tuning methods for the temperature control of an air heater. He assigned the best identification method as the Skogestad's method [5]. Canojiva and Meshram (2012) presented a tuning method of a PI controller for DC motor control using particle swarm optimization, Ziegler-Nichols tuning and Modified Ziegler-Nichols method. Their objective was to minimize the rise time, settling time and maximum overshoot. They concluded that their tuning method was more efficient and robust when compared with the other two methods [6].

Airikka (2014) considered the stability and robustness of a predictive PI controller without additional filter for time-invariant system He derived PI controller tuning rules for a first-order plus dead time and integrating plus dead time systems [7]. Irshad and Ali (2018) reported optimal tuning rules for PI/PID controllers for stable and integrating first-order inverse response processes. They used ISTE; IST^2E and IST^3E performance indices minimized using particle swarm optimization to tune the controllers. They concluded that the IST^3E criterion produced less undershoot better set-point tracking and disturbance rejection [8].

Cong, Juh, Trong and Ba (2019) used the chemical reaction optimization algorithm to tune a PI controller. They used a doubly fed induction generator as a process. They used the chemical reaction optimization (CRO) to optimize the PI controller parameters and presented a comparison with the Ziegler-Nichols tuning method [9]. Ahmadi, Nikravesh and Amani (2020) proposed a tuning method for PI/PID controllers within the filtered Smith predictor (FSP) configuration to deal with time delay processes. The proposed PI/PID controller was designed and tuned using the IMC principal. They compared with other tuning methods to explore the effectiveness of their proposed tuning method [10].

Veronesi and Visioli (2022) presented tuning rules for PI controllers for load disturbance rejection. They used an IAE index subject to a constraint on the selected maximum sensitivity. They also considered self-regulating and non-self-regulating processes. They presented simulated examples to clarify the application of their tuning technique [11]. Lai and Hoo (2023) proposed an anti-windup controller with a semi-decoupled tuning as design. They compared the performance of the proposed controller with the conventional PI controller and the steady-state integral PI controller [12].

II. THE CONTROLLED PROCESS

The controlled is a second-order-like linear process having the transfer function, $G_p(s)$ given by:

$$G_p(s) = \omega_n^2 / (s^2+2\zeta\omega_n s+\omega_n^2) \qquad (1)$$

Where:

ω_n = process natural frequency (rad/s).

ζ = process damping ratio.

The present study presents the tuning of a PI controller for process parameters in the range:

$$1 \le \omega_n \le 9 \quad \text{rad/s}$$
$$0.2 \le \zeta \le 2 \qquad (2)$$

The damping range covers underdamped, critically damped and overdamped second-order-like processes.

III. PROCESS CONTROL USING A PI CONTROLLER

- A conventional PI controller has the transfer function [13]:

$$G_c(s) = K_{pc}+(K_i/s) \qquad (3)$$

Where:

K_{pc} = proportional gain.

K_i = integral gain.

The block diagram of the feedback control system comprising the PI controller and the second-order process with both reference and disturbance inputs is shown in Fig.1.

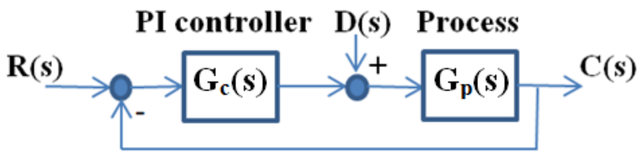

Fig.1 Block diagram of a PI controlled process.

The transfer function of the control system using Fig.1 depends on the input as follows:

- *With reference input R(s), $M_R(s)=C(s)/R(s)$ is:*

$$M_R(s) = N_R(s) / D_R(s) \qquad (4)$$
$$N_R(s) = \omega_n^2 K_{pc} s+\omega_n^2 K_i$$
$$D_R(s) = s^3+2\zeta\, \omega_n s^2+\omega_n^2(1+K_{pc})s+ \omega_n^2 K_i$$

- *With disturbance D(s), MD(s) = C(s)/D(s) is:*

$$M_D(s) = N_D(s) / D_D(s) \qquad (5)$$
$$N_D(s) = \omega_n^2 s$$
$$D_D(s) = s^3+2\zeta\, \omega_n s^2+\omega_n^2(1+K_{pc})s+ \omega_n^2 K_i$$

IV. PI CONTROLLER TUNING

The PI controller is tuned as follows:

- The unit step time response of the control system, c(t for a reference input is obtained using Eq.4 and the '*step*' command of MATLAB [14].

- The error signal e(t) of the control system is assigned as: 1 – c(t) for a control system having the block diagram in Fig.1 and a unit step input.
- The integral of time multiplied by absolute error (ITAE) performance index [15] is minimised by the MATLAB optimization toolbox [16].
- The error function incorporates the controller parameters K_{pc} and K_i.
- Minimizing the error function ITAE reveals the optimal gain parameters of the controller.
- The controller tuning technique is applied for process parameters in the range assigned in Eq.2.
- The PI controller tuning results are tabulated in Tables 1 through 4 below:

Table 1: PI controller tuning for ω_n = 1 rad/s.

Process damping ratio (ζ)	Proportional gain (K_{pc})	Integral gain (K_i)
0.2	0.099888	0.089634
0.4	0.0996716	0.177708
0.6	0.0690720	0.297933
0.8	0.073701	0.163479
1.0	1.485630	0.784865
1.20	0.509197	0.469013
1.40	0.599766	0.426359
1.60	0.745344	0.393580
1.80	0.899808	0.341125
2.00	1.2097560	0.433734

Table 2: PI controller tuning for ω_n = 3 rad/s.

Process damping ratio (ζ)	Proportional gain (K_{pc})	Integral gain (K_i)
0.2	0.0501178	0.235230
0.4	0.0499404	0.3457716
0.6	0.0245584	0.6051380
0.8	0.2652940	1.1349520
1.0	0.2508132	1.0314690
1.20	0.4479620	1.0749120
1.40	0.570458	1.0591530
1.60	0.729760	1.0477130
1.80	0.928295	1.0671730
2.00	1.318352	1.0768140

Table 3: PI controller tuning for ω_n = 5 rad/s.

Process damping ratio (ζ)	Proportional gain (K_{pc})	Integral gain (K_i)
0.2	0.050073	0.369353
0.4	0.045058	1.020900
0.6	0.309840	1.482270
0.8	0.257000	1.050800
1.0	0.233473	1.062500
1.20	0.807875	1.598710
1.40	0.591261	1.339050
1.60	0.718110	1.141394
1.80	0.909060	1.174830
2.00	1.297030	1.234896

Table 4: PI controller tuning for ω_n = 7 rad/s.

Process damping ratio (ζ)	Proportional gain (K_{pc})	Integral gain (K_i)
0.2	0.001120	0.296980
0.4	0.142410	1.032200
0.6	0.288050	1.054600
0.8	0.221270	1.169750
1.0	0.226835	1.087689
1.20	0.866370	2.002970
1.40	0.592740	1.890670
1.60	0.618440	1.494560
1.80	0.880905	1.241436
2.00	1.344750	2.768310

Table 5: PI controller tuning for ω_n = 9 rad/s.

Process damping ratio (ζ)	Proportional gain (K_{pc})	Integral gain (K_i)
0.2	0.001000	0.342960
0.4	0.118630	0.353360
0.6	0.267110	0.586410
0.8	0.212845	1.035140
1.0	0.259057	1.044280
1.20	0.838230	1.080700
1.40	0.578250	1.446780
1.60	0.607776	1.095430
1.80	0.880100	1.121940
2.00	1.196390	2.517330

V. CASE STUDY

Consider a second-order-like process having the parameters:

- Natural frequency: 3 rad/s
- Damping ratio: 0.4

The process is to be controlled by a PI controller. Tune the controller using the technique presented in this research work and compare with the minimum ITAE standard forms technique.

- Using the tuned PI controller parameters in Table 2, the optimal controller parameters are:

$K_{pc} = 0.0499404$, $K_i = 0.3457716$ (6)

- Using the minimum ITAE standard forms [17]:
  - The optimum characteristic equation for a first-order numerator and third-order denominator of the control system transfer function is [17]:

    $s^3+1.75\omega_0 s^2+3.25\omega_0^2 s+\omega_0^3 = 0$ (7)
  - Comparing the characteristic equation of the closed loop control system in Eqs.4 and 7 reveals the PI controller parameter as:

    $K_{pc} = -0.32090$, $K_i = 0.28656$ (8)
  - The unit step time response of the control systems for reference input tracking using the control system transfer function in Eq.4 and the tuning controller gains in Eqs.6 and 8 is generated by the step command of MATLAB [14] and shown in Fig.2. The control system has the time-based characteristics:
    - Maximum overshoot: zero compared with 2.52 % for the minimum ITAE tuning technique.
    - Maximum undershoot: zero compared with 15.58 % for the minimum ITAE tuning technique.
    - Settling time: 9.56 s compared with 6.55 s for the minimum ITAE tuning technique.
  - The unit step time response of the control systems for disturbance input tracking using the control system transfer function in Eq.5 and the tuning controller gains in Eqs.6 and 8 is generated by the step command of MATLAB [14] and shown in Fig.3. The control system has the time-based characteristics:

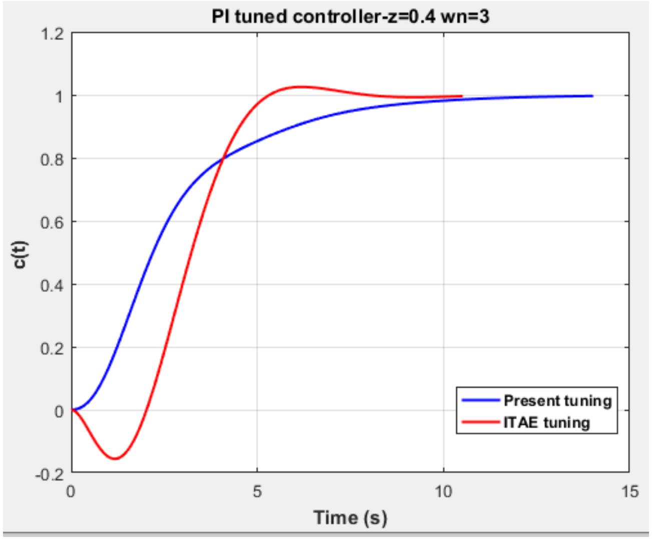

Fig.2 Step time response for reference input.

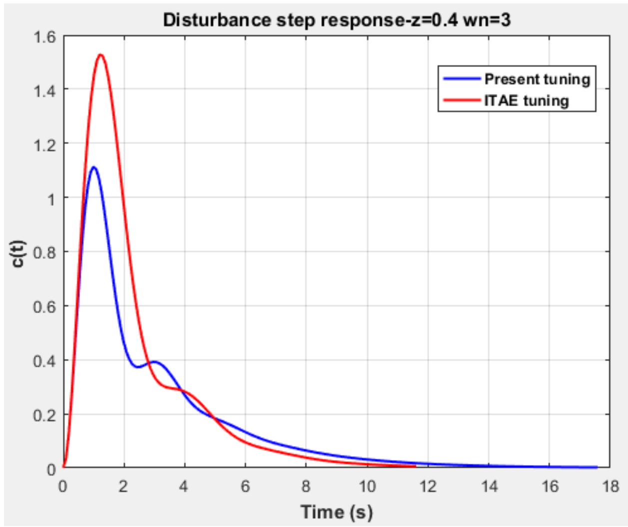

Fig.3 Step time response for disturbance input.

- Maximum time response: 1.113 s compared with 1.527 s for the minimum ITAE tuning technique.
- Time of maximum time response: 1.0 s compared with 1.22 s for the minimum ITAE tuning technique.
- Settling time: 16 s compared with 12 s for the minimum ITAE tuning technique.

VI. CONCLUSIONS

- The research work presented in this research paper handled the tuning of PI controllers used to control second-order-like processes.
- The controlled process had natural frequency in the range from 1 to 9 rad/s and a damping ratio from 0.2 to 2 (underdamped, critically damped and overdamped second-order-like processes).
- The PI controller was tuned using the MATLAB optimization toolbox with an ITAE performance index aiming at providing a stable control system and good dynamic performance.
- The performance of the control system incorporating the PI controller tuned using the approach presented in the paper was compared with another tuning technique (minimum ITAE standard forms).
- The PI controller tuning results were presented in five tables for the process damping rations: 1, 3, 5, 7 and 9 rad/s against the process damping ratio values: 0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.4, 1.6, 1.8 and 2.
- A case study was presented as application on the PI controller tuning results to examine its effectiveness in terms of the time-based characteristics: maximum overshoot, minimum overshoot and settling time.
- The step time response of the closed-loop control system was investigated for both reference and disturbance inputs and their characteristics were compared.
- The present tuning technique for the PI controller succeeded to eliminate completely any overshoot or undershoot in the step time response for reference input tracking.
- In the case study, the PI controller with the present tuning technique succeeded to reduce the maximum time response for the disturbance input by 27.1 % and settle to zero after 16 seconds.
- It succeeded also to reduce the time of maximum step time response due to the disturbance input by 18 %.

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