RESEARCH ARTICLE OPEN ACCESS

Joint-Sparsity and Total Variation approaches for Hyperspectral image unmixing

K. Ramudu Guide[1], E. Mounika Lakshmi[2], S.Mohammad anifh[3], S. Sajjad Basha[4]
1.(Department of ECE, Annamacharya Institute of Technology & Sciences, Rajampet Email.com:ramudu.kamanuri@gmail.com)
2. (Department of ECE, Annamacharya Institute of Technology & Sciences, RajampetEmail.com: emounikalakshmi@gmail.com)3. (Department of ECE, Annamacharya Institute of Technology & Sciences, RajampetEmail.com:mohammadshaik687@gmail.com)
4.(Department of ECE, Annamacharya Institute of Technology & Sciences, Rajampet Email.com: sajjad.basha22@gmail.com)

Abstract:

Hyperspectral image means it is a collection of hundreds of spectral bands between ultra violets to infrared ray wavelengths for the same area on the surface of the earth. It has a wide range of applications like environment monitoring, military surveillance, mineral exploration, astronomy etc. Hyperspectral images are often corrupted by several noise during the image acquisition, image transmission etc. this highly degrades the visual quality of hyper spectral images. So, by using joint-sparsity and total variation we are going to show noisy image as denoisy image and its PSNR value as output. Here to optimize the denoising algorithm split-Bergman technique is use.

I. INTRODUCTION:

In image processing hyperspectral imaging is fastest developing research area. Hyperspectralunmixing is an important and challenging problem in remote sensing. In hyperspectral image the pixel which contains only one object called as pure pixel. The pixel which contains more than one object or material is called mixed pixel. Hyperspectral images have high resolution. Those are often corrupted by several kinds of noise like Gaussian noise, sparse noise. Gaussian noise occurs during dark current, or lighting or sensor noise. Whereas sparse noise occur due to sensors or temperature changes in the environment. These decrease the quality of the image. In order to remove noise we are going to use our proposed algorithm. Our proposed algorithm is based on linear mixing model for unmixing. Linear mixing model is mathematical written as

$$P_i = \sum_{j=1}^{n} (R_{ij} \cdot F_j) + E_i$$

Whereas$i = 1,\ldots,m$(number of bands)

$j = 1,\ldots,n$(number of endmembers)

P_i= spectral reflectance of the ith spectral band of a pixel.

R_{ij}= known spectral reflectance of the jth component.

F_j= the fraction coefficient of the j^{th}component within the pixel.

E_i= error for the ith spectral band.Expanding (1) to all spectral bands gives the matrix form of the linear unmixing equation is written as

P= RF+E.

Where

$$P=\begin{bmatrix}P_1\\P_2\\\vdots\\P_m\end{bmatrix};\quad R=\begin{bmatrix}R_{11}&R_{12}&\dots&R_{1n}\\R_{21}&R_{22}&\dots&R_{2n}\\\vdots&\vdots&\vdots&\vdots\\R_{m1}&R_{m2}&\dots&R_{mn}\end{bmatrix};\quad F=\begin{bmatrix}F_1\\F_2\\\vdots\\F_n\end{bmatrix};\quad E=\begin{bmatrix}\\\\\\\end{bmatrix}$$

Generally, (1) has to follow two constraints of non-negativity and sum-to-one, based on physical considerations

$\sum f_i=1,\quad f_j\geq 0,\ j=1,2,3,..,n.$

However other non-linear mixing models like pixel purity pixel (PPI) and N-FINDER which require pure pixel image. We are not using these models.Joint sparsity generally mean as group of thin elements. Sparsity is a powerful tool in the signal processing. It is typically useful in inverting the linear system which is undetermined form.

$$Y= Ax \qquad (1)$$

Whereas Y is P-dimensional measurement vector and x is a Q-dimensional sparse signal vector. A is in matrix form.Total variation denoising means smoothing or median filters the image by removing the noise at the same time smooth away edges to greater or lesser degree. This technique is used in digital image processing. Total variation denosing is also called as total variation regularization. The regularization parameter plays a vital role in the denoising process. When there is no smoothing the result is the same as minimizing the sum of square.Soft thresholding means suppressing low correlations in a continuous manner rather than the discontinuous thresholding used in constructing unweight networks. All these are done by Split-Bergman technique that is in next page.

II.Literature survey:

In previous algorithms only some noise is removed. So, if we want to remove noise we have to use many algorithms. For example if we consider *hyperspectral image restoration using low rank matrix recovery and neutral network*(LRMR) it reduces the Gaussian noise, dead pixels or line stripsin such a way we have many existing algorithms onlyone or two problems can solve but with the help of proposed algorithm i.e.,Split-Bergman technique is used solve L1-regularization; compressed sensing problem arises in Magnetic resonance imaging, total variation and some other regularization.

III. Proposed formulation:

A hyperspectral images may contain a mixture of Gaussian and sparse noise. Let us consider the mixednoise model for unmixing and account for both types of noise. The usual unmixing model is $\min_a \|y-Ma\|_2^2$ subject to $\|a\|_0\leq k$ (2)

can be extended as

$$Y = MA+ S + G,\ A\geq 0 \ (3)$$

whereas$S$ and G represents sparse and Gaussian noise. The above noise model assumes both Gaussian and sparse noise to be additive noise. Sparse noise means it consist of horizontal or vertical line strips, shot noise or any impulse noise present in a hyperspectral image. They corrupt few pixels in a hyperspectral image. By using this model, we can formulate the unmixing problem as

$$\min_{A,S}\|Y-MA-S\|_F^2+\lambda_1\|A\|_{2,1}+\lambda_2\|S\|_1 \quad (4)$$

In the above equation first term is data fidelity which is equivalent to minimizing the variance of Gaussian noise $G = Y - MA - S$. First regularization term is an L21-norm minimization on abundance matrix Awhich is also called joint sparse regularization term. This term is based on the observation that in most hyperspectral images, a fewer endmembers are present when compared with the available endmembers. This observation is mathematically written as joint-sparse regularization on matrix A with few non-zero rows, but each non-zero row is allowed to be dense. The second regularization term corresponds to minimizing $L1$-norm of sparse noise matrix S. Here, $L1$-norm is minimized due to modeling assumption that sparse noise affects few pixels in the image. As an alternative unmixing model, we can also exploit the fact that most natural images are piece-wise smooth for example, if there are some vegetation pixels in the image the nearby pixels are also likely to be vegetation pixels. Therefore, the abundance maps can be considered as piecewise smooth. The piecewise smoothness can be modeled as total-variation regularization

$$\underset{A,S,P,Q}{\text{minimize}}\quad \|Y-MA-S\|_F^2+\lambda_1\|P\|_1+\lambda_2\|Q\|_{2,1}+\lambda_3\|S\|_1$$

(5)

Whereas, $\nabla$is two-dimensional total-variation operator that applies total variation along both horizontal and vertical directions on a two-dimensional image. The ∇operatoris applied on AT because each abundance map is a long rows of A. In this work, we propose to simultaneously exploit both the joint-sparsity as well as spatial smoothness of the abundance maps in the light of generic noise model. Thus the proposed hyperspectralunmixing problem formulation can be expressed as

$$\min_{A,S}\|Y-MA-S\|_F^2+\lambda_1\|A^T\|_1+\lambda_2 A_{2,1}+\lambda_3 S_1. \quad (6)$$

Whereas, $\lambda 1$, $\lambda 2$, and $\lambda 3$ are regularization parameters which are corresponding to total-variation, joint-sparsity and sparse noise terms. These three models

in (4)-(6) estimate sparse noise S as a byproduct of the proposed formulations. Let $Y = MA$ be the clean image then we can get denoised image $\hat{Y} = M\hat{A}$ where $\hat{A}$ is the estimated abundance maps by solving (6). Along with generic noise model (4), we have both joint-sparsity as well as piecewise smoothness of abundance maps. We are not aware of any efficient algorithm to solve (6); therefore, in the next section, we briefly describe how to solve this problem using the split-Bregman based technique.

IV. Proposed algorithm:

The split-Bregmanalgoritm can solve certain convex optimization problems and this algorithm is a row-action method accessing constraint functions one by one. The split-Bregmanapproach can be utilized to derive the algorithm for solving (6). The split-Bregmanapproach is suitable to solve (6) because it has been designed to handle multiple regularization terms. The variable A is not separable in (6); therefore, we utilize auxiliary variables P and Q to make the problem separable. Set $P = \nabla AT$ and $Q = A$, then we get following constrained problem:

minimize $\quad Y - MA - S^2_F + \lambda_1 P_1 + \lambda_2 Q_{2,1} + \lambda_3 S_1$ (7)

subject to $\quad P = \nabla AT$ (7a)

$Q = A$ (7b)

This problem can be rewritten into unconstrained form by using two Bregman variables $B1$ and $B2$ to getminimize $Y - MA - S^2_F + \lambda_1 P_1 + \lambda_2 Q_{2,1} + \lambda_3 S_1$

A,S,P,Q

$+\mu_1 P - \nabla A^T - B_1{}^2_F + \mu_2 Q - A - B_2{}^2_F$ (8)

Whereas$B1$ and $B2$ are updated as

$$B1 = B1 + \nabla AT - P$$

$$B2 = B2 + A - Q.$$

Above problem is separable in each variable therefore can be written the sub-problems as

$$\text{P1:} \quad \min_P \ \mu_1 \|P - \nabla A^T - B_1\|_F^2 + \lambda_1 \|P\|_1$$

$$\text{P2:} \quad \min_Q \ \mu_2 \|Q - A - B_2\|_F^2 + \lambda_2 \|Q\|_{2,1}$$

$$\text{P3:} \quad \min_S \ \|Y - MA - S\|_F^2 + \lambda_3 \|S\|_1$$

$$\text{P4:} \quad \min_A \ \|Y - MA - S\|_F^2 + \mu_1 \|P - \nabla A^T - B_1\|_F^2 + \mu_1 \|Q - A - B_2\|_F^2$$

each of these problems can be solved iteratively by using Slit-Bregman iteration with Bregman variables updated in k^{th}iteration as

$$B_1{}^{k+1} = B_1{}^k + \nabla (A^k)^T - P^k$$

$$B_2{}^{k+1} = B_2{}^k + A^k - Q^k.$$

The problems $P1$ and $P3$ are of the form

The problems $P1$ and $P3$ are of the formwhich can be solved by using soft-thresholding operation

$$\hat{x} = \text{SoftTh}(y, \lambda) = \text{sign}(y) \times \max\left\{0, |y| - \frac{\lambda}{2}\right\} \quad (9)$$

The problem $P2$ can be solved This is a _2-norm shrinkage operation on each row $q(i)$ $\forall i = 1, 2, \ldots, e$, of matrix Q. The L_2-norm shrinkage problem is

$$\min_x \|y - x\|_2^2 + \lambda \|y\|_2 \quad (10)$$

whose solution is given by

$$\hat{x} = \text{Shrink}(y, \lambda) = \max\left\{\|y\|_2 - \frac{\lambda}{2}, 0\right\} \odot \frac{y}{\|y\|_2}$$

WhereasOrepresent element by element multiplication operationwith the assumption that $0 \times 0/0 = 0$. The problem $P4$ is a differentiableconvex optimization problem. After differentiatingwe get following linear system of equations with variable A:

$$M^T MA + \mu_1 A \nabla^T \nabla + \mu_2 A = M^T (Y - S) + \mu_1 (P^T - B_1^T) \nabla + \mu_2 (Q - B_2) \quad (11)$$

The below system of linear equation is large and sparse whose solution can be approximated using algorithms such as LSQR Algorithm 1 outlines the steps of proposed jointly sparse and total-variation regularized hyperspectralunmixing algorithm using the split-Bregman approach.

$$\Psi a = vec(M^T (Y - S)) + \alpha \quad \text{where}$$

$$\Psi = (I_e \otimes M^T M) \mu_1 (\nabla^T \nabla \otimes I_e) + \mu_1 I_{pe}$$

$$\alpha = \mu_1 (P^T - B_1^T) \nabla + \mu_2 (Q - B_2)). \quad (12)$$

We use the acronym JSTV for the proposed joint-sparsity and total variation based unmixing method. By setting $\lambda 1 = 0$, we can derive the solution of (6) which we refer as split-Bregman algorithm-based joint-sparse regularized (SBJS) unmixing algorithm. Similarly, $\lambda 2 = 0$ results in an algorithm that solves (5) which we refer as split-Bregman algorithm based total variation regularized (SBTV) unmixing algorithm.

V. Proposed work:

The flow chart is shown in below diagram as

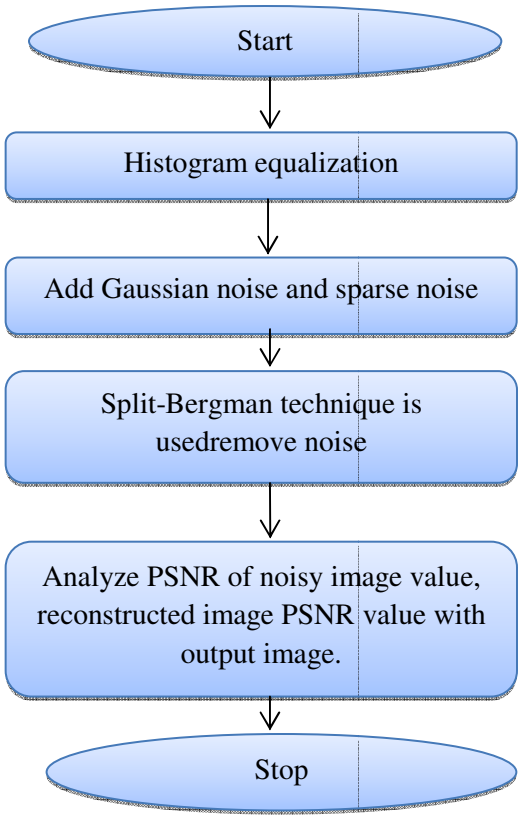

Figure1:flow chart for proposed work

Some important steps are written below

Step1: Code is developed to load the image to access in the MATLAB.

Step2: Histrogram equalization code is developed.

Step3: Code is developed to add gaussian and sparse noise to the image.

Step4: Code is developed to split-Bergman iterations.

Step5: Code is developed to print PSNR of noisy image, elapsed time, reconstructed abundance, reconstructed PSNR image value.

Result:

In the below image we can see five different original and reconstructed images as output.

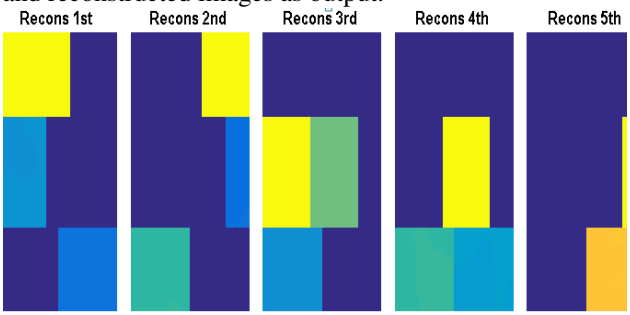

Figure2:Five different reconstructed images

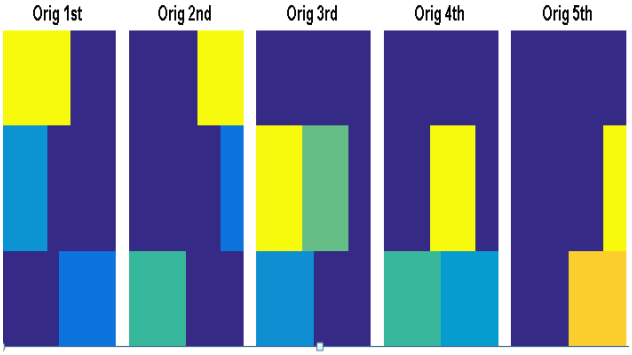

Figure3:Five different original images

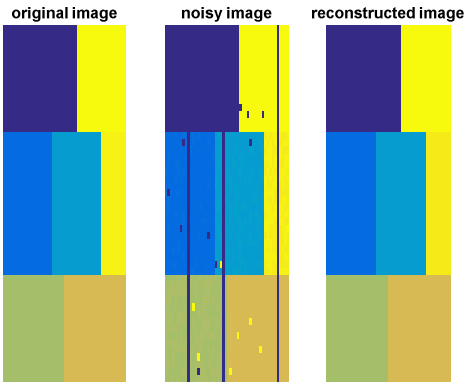

Figure4: Reconstructed image by noisy image

In figure4 to original image noise is added and shown as noisy image. Then from noisy image reconstructed image is shown.

VI. Tabular column:

S. No	Iterations	PSNR of noisy image	Elapsed time	Reconstructed abundance	Reconstructed image PSNR
1.	0	8.997	0.2742	7.0289	2.935
2.	25	9.479	27.072	41.502	43.661
3.	50	8.997	50.923	69.772	72.910
4.	100	9.479	98.81	78.493	77.384
5.	150	9.755	149.1	80.316	78.123
6.	200	9.329	199.96	81.219	78.272

VII. Graph:

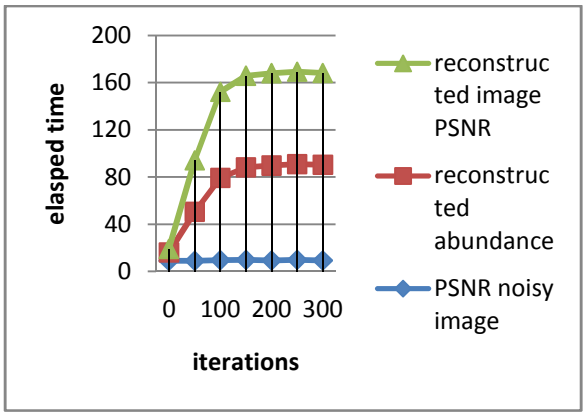

Figure5: Graph by taking iterations VS elapsed time

VIII. Conclusion:

Hyperspectral images play vital role in remote sensing. These are often corrupted by several noise those are removed with the help of split-Bergman technique. This technique gives great accuracy.The images which is used in this algorithm is taken from USGS spectral library.

Reference:

[1]J. M. Bioucas-dias, A. Plaza, N. Dobigeon, M. Parente, Q. Du, P. Gader, and J. Chanussot, "Hyperspectralunmixing overview: Geometrical, sta-tistical, and sparse regression-based approaches," *IEEE J. Sel. TopicsAppl. Earth Observ. Remote Sens.*, vol. 5, no. 2, pp. 354–379,Apr. 2012.

[2] A. Ertürk, "Enhanced unmixing-based hyperspectral image denoisingusing spatial preprocessing," *IEEE J. Sel. Topics Appl. Earth Observ.Remote Sens.*, vol. 8, no. 6, pp. 2720–2727, Jun. 2015.

[3] J. M. Bioucas-dias, A. Plaza, N. Dobigeon, M. Parente, Q. Du, P. Gader,and J. Chanussot, "Hyperspectralunmixing overview: Geometrical, statistical,and sparse regression-based approaches," *IEEE J. Sel. TopicsAppl. Earth Observ. Remote Sens.*, vol. 5, no. 2, pp. 354–379,Apr. 2012.

[4] R. Heylen, M. Parente, and P. Gader, "A review of nonlinear hyperspectralunmixing methods," *IEEE J. Sel. Topics Appl. Earth Observ. RemoteSens.*, vol. 7, no. 6, pp. 1844–1868, Jun. 2014.[5] J. W. Boardman, "Automating spectral unmixing of AVIRIS data usingconvex geometry concepts," in *Proc. Summ. 4th Annu. JPL AirborneGeosci. Workshop*, 1993, pp. 11–14.[6] D. Cerra, M. Rupert, and P. Reinartz, "Noise reduction in hyperspectralimages through spectral unmixing," *IEEE Geosci. Remote Sens. Lett.*,vol. 11, no. 1, pp. 109–113, Jan. 2014.[7] H. Zhang,W. He, L. Zhang, H. Shen, and Q. Yuan, "Hyperspectral imagerestoration using low-rank matrix recovery," *IEEE Trans. Geosci. RemoteSens.*, vol. 52, no. 8, pp. 1–15, Aug. 2014.

[8] T. Goldstein and S. Osher, "The split Bregman method for L1-regularizedproblems," *SIAM J. Imaging Sci.*, vol. 2, no. 2, pp. 323–343, Jan. 2009.

[9] M.-d. Iordache, J. M. Bioucas-dias, and A. Plaza, "Total variation spatialregularization for sparse hyperspectralunmixing," *IEEE Trans. Geosci.Remote Sens.*, vol. 50, no. 11, pp. 4484–4502, Nov. 2012.

[10] W. Yin, S. Osher, D. Goldfarb, and J. Darbon, "Bregman iterative algorithmsfor L1-minimization with applications to compressed sensing,"*SIAM J. Imaging Sci.*, vol. 1, no. 1, pp. 143–168, 2008.

[11] USGS. *Spectral Library* [Online]. Available: http://speclab.cr.usgs.gov/ spectral-lib.html, accessed on Nov. 30, 2015.

[12] S. Grupo de InteligenciaComputacional, Universidad del Pas Vasco /EuskalHerrikoUnibertsitatea (UPV/EHU). *Hyperspectral ImagerySynthesis (EIAs) Toolbox* [Online]. Available: http://www.ehu.es/ccwintco/index.php/Hyperspectral Imagery Synthesis tools forMATLAB, accessed on Nov. 30, 2015.

[13] AVIRIS. *Jasper Image* [Online]. Available: http://www.cossa.csiro.au/ hswww/Overview.htm, accessed on Nov. 30, 2015.