**TOA and DOA Joint Estimation Using Successive MUSIC Algorithm in IR-UWB Positioning Systems**

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**Abstract:**

A Wireless Sensor Network (WSN) is an autonomous and self-organizing network without any pre-established infrastructure which offers many advantages in military applications and emergency areas. Source Localization is one of the important monitoring tasks of the WSN. It provides the accurate position of the source using various positioning technologies. In this paper an Impulse Radio Ultra wideband (IR-UWB) positioning system with a two-antenna receiver is used to estimate the Time of arrival (TOA) and Direction of arrival (DOA) positioning parameters. A two dimensional (2D) multiple signal classification (MUSIC) algorithm is used to estimate these parameters but it has much higher computational complexity and also requires 2D spectral peak search. A Successive Multiple signal classification (MUSIC) algorithm is proposed in this paper which estimates the parameters jointly and gets paired automatically. It avoids the two dimensional peak searches and reduces the complexity compared to the existing methods 2D-MUSIC, Root-MUSIC, Matrix Pencil algorithm, Propagator method and Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) algorithm.

**Keywords**--Time of arrival (TOA), Direction of arrival (DOA), Impulse Radio Ultra Wideband (IR-UWB), Multiple Signal Classification (MUSIC).

**1. INTRODUCTION**

Position information is one of the key requirements for a Wireless Sensor Network (WSN) [1] to function as intended. An Impulse Radio Ultra Wideband (IR-UWB) positioning system provides an excellent means for wireless positioning [2] due to its high resolution capability in the time domain. UWB systems refer to the systems with very large bandwidth which offers several advantages including high data rate, low complexity, low cost implementation, low power consumption, resistance to interference, covert transmission and high anti-multipath effects which has made UWB widely used in radars, localization, tracking, sensor networks, positioning and imaging [3],[4]. Short range high speed communication is the most popular application of the UWB. Time of arrival (TOA) and Direction of arrival (DOA) [5] are the key signal parameters to estimate in the positioning systems.

TOA estimation techniques are classified into two categories: cross-correlation based and super resolution based. The cross correlation based estimation methods cannot fulfill the required accuracy for many applications because it provides very low resolution and limited bandwidth. To achieve the desired accuracy super resolution techniques have been proposed for the TOA estimation [6],[7]. These techniques offers high resolution spectral estimation with good accuracy and are applied after the estimated channel impulse response is transformed to frequency domain.

DOA estimation [8],[9] techniques are classified into three methods: spectral based, parametric based and sub-space based methods. In this paper sub-space based DOA estimation methods are considered for UWB signals. In the sub-space based DOA estimation methods there are several algorithms which are MUSIC algorithm [10],[11], Root-MUSIC algorithm, ESPRIT algorithm, Matrix pencil algorithm, Propagator method.
In this paper two dimensional (2D) multiple signal classification (MUSIC) algorithm is employed first to estimate the Time of Arrival (TOA). This 2D-MUSIC algorithm is here extended to estimate jointly the Time of Arrival (TOA) and the Direction of Arrival (DOA) in Impulse Radio-Ultra Wideband (IR-UWB) systems [12],[13]. It solves the pairing problem but renders much higher computational complexity. To reduce the computational complexity a Successive MUSIC algorithm is proposed in this paper for joint TOA and DOA estimation in IR-UWB system with a two-antenna receiver. This proposed algorithm firstly gets the initial estimates of TOA corresponding to the first antenna and then simplifies the 2D global search into successive one-dimensional (1D) searches through which the TOAs in the two antennas are estimated. Then the DOA estimates are obtained from the difference of the TOAs between the two antennas.

The proposed Successive MUSIC algorithm has the following advantages:

- Parameters are automatically paired.
- Lower computational complexity.
- Better Parameter estimation performance.

II. SYSTEM MODEL

The received UWB signal of an IR-UWB system can be expressed as the convolution of transmitted signal \( s(t) \) and impulse response \( h^{(k)}(t) \) which is shown as:

\[
 r^{(k)}(t) = s(t) * h^{(k)}(t) + n^{(k)}(t) \tag{1}
\]

Where \( * \) denotes the convolution and \( n^{(k)}(t) \) represents the additive Gaussian white noise in the \( k \)th cluster.

The transmitted signal \( s(t) \) is given as:

\[
 s(t) = \sum_{j=-\infty}^{+\infty} \sum_{m=0}^{N_c-1} b_j c_m w(t - jT_s - mT_c) \tag{2}
\]

With \( w(t) \) denoting the received UWB pulse where \( T_c \) and \( T_s \) represents the symbol and chip duration and \( N_c \) the number of pulses representing one information symbol. The direct sequence binary phase shift keying (DS-BPSK) modulation is assumed, with representing \( b_j \in \{ -1, +1 \} \) as the bi-phase modulated data sequence, \( c_m \in \{ -1, +1 \} \) as the user-specific code sequence. The UWB channel is expressed in a period composed of \( K \) clusters and \( L \) multiple paths included in each cluster.

The UWB channel impulse response in the \( k \)th cluster can be expressed by

\[
 h^{(k)}(t) = \sum_{l=1}^{L} \alpha^{(k)}_l \delta(t - \tau_l) \tag{3}
\]

Where \( \delta(t) \) is the Dirac delta function, \( \tau_l \) is the propagation delay of the \( l \)th path relative to the \( k \)th cluster and \( \alpha^{(k)}_l \) represent random complex fading amplitude.

Transforming the received signal to the frequency domain, we can obtain

\[
 R^{(k)}(\omega) = S(\omega)H^{(k)}(\omega) + N^{(k)}(\omega)
\]

\[
 = \sum_{l=1}^{L} \alpha^{(k)}_l S(\omega)e^{-j\omega\tau_l} + N^{(k)}(\omega) \tag{4}
\]

where \( R^{(k)}(\omega) \), \( S(\omega) \), \( H^{(k)}(\omega) \), \( N^{(k)}(\omega) \) are Fourier transform of \( r^{(k)}(t) \), \( s(t) \), \( h^{(k)}(t) \), \( n^{(k)}(t) \) respectively. Then the frequency domain signal model can be obtained by sampling equation (4) at \( \omega_n = m\Delta \omega \) for \( m=0, 1, \ldots, M-1 \) and \( \Delta \omega = 2\pi/M(M > L) \), and rearranging the frequency samples \( R^{(k)}(\omega) \) into vector \( r^{(k)} = [R^{(k)}(\omega_0), \ldots, R^{(k)}(\omega_{M-1})]^T \in \mathbb{C}^{M\times1} \) as

\[
 r^{(k)} = SE_{\tau}a^{(k)} + n^{(k)} \tag{5}
\]

Where \( S \in \mathbb{C}^{M\times M} \) is a diagonal matrix whose components are the frequency samples \( S(\omega_n) \) and \( E_{\tau} = [a_1, \ldots, a_L] \in \mathbb{C}^{M\times L} \) is a delay matrix with the column vectors being \( a_l = [1, e^{-j\omega_1\tau_l}, \ldots, e^{-j(M-1)\omega_1\tau_l}]^T \) for \( l = 1, 2, \ldots, L \). The channel fading coefficients of \( k \)th cluster are arranged in the vector \( a^{(k)} = [\alpha^{(k)}_1, \ldots, \alpha^{(k)}_L]^T \in \mathbb{C}^{M\times1} \), and the noise samples in vector \( n^{(k)} = [N^{(k)}(\omega_0), \ldots, N^{(k)}(\omega_{M-1})]^T \in \mathbb{C}^{M\times1} \). Here \( M \) refers to the discrete Fourier transform (DFT) of the UWB signal.

III. TOA AND DOAJOINT ESTIMATION

UWB sources are considered which are located in the far field from the array consisting of two antennas, which is shown in Fig. 1. We assume the TOAs associated to the \( l \)th path in the antenna 1 and antenna 2 for \( l = 1, 2, \ldots, L \) as \( \beta_l \) and \( \gamma_l \).
A. 2D-MUSIC Algorithm for TOA Estimation

A matrix $Z \in \mathbb{C}^{2M \times k}$ is taken as shown below

$$Z = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} SE_\beta \\ SE_\gamma \end{bmatrix} F + \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} \quad (12)$$

Let $A(\beta, \gamma) = \begin{bmatrix} SE_\beta \\ SE_\gamma \end{bmatrix} E^H \in \mathbb{C}^{2M \times L}$ and $N = \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} \in \mathbb{C}^{2M \times K}$. Hence the matrix $Z$ can be written as $Z = A(\beta, \gamma) F + N$. The covariance matrix $\hat{R}$ is taken as $\hat{R} = ZZ^H / K$. Eigenvalue decomposition is performed for the covariance matrix $\hat{R}$ to get the signal subspace $\hat{U}_N \in \mathbb{C}^{2M \times L}$ and noise subspace $\hat{U}_N \in \mathbb{C}^{2M \times (2M-L)}$. Then we can establish the 2D-MUSIC spatial spectrum function in this form

$$P_{2D-MUSIC}(\beta, \gamma) = \frac{1}{a(\beta, \gamma)^H \hat{U}_N \hat{U}_N^H a(\beta, \gamma)} \quad (13)$$

Where $a(\beta, \gamma)$ is the column vector of matrix $A(\beta, \gamma)$. Hence we take the $L$ largest peaks of $P_{2D-MUSIC}(\beta, \gamma)$ as the estimates of the TOAs. Obviously, the multipath delays in the two antennas can be accurately estimated via 2D-MUSIC, in which the exhaustive 2D search, however, is normally inefficient due to high computational cost. In the following subsections, we present another MUSIC algorithm, which qualifies the TOA estimation just through the 1D search.

B. Successive MUSIC Algorithm for 2D-TOA Estimation

In this paper Successive MUSIC algorithm for angle estimation in multiple-input multiple-output radar is extended to the UWB parameter estimation. Consider $X_1$, the received signal of antenna 1 in frequency domain, and perform eigenvalue decomposition of the covariance matrix $\hat{R}_1 = X_1 X_1^H / K$ to get the signal subspace and noise subspace as

$$\hat{R}_1 = \hat{E}_S \hat{\Lambda}_S \hat{E}_S^H + \hat{E}_N \hat{\Lambda}_N \hat{E}_N^H \quad (14)$$

Where $\hat{\Lambda}_S$ is an $L \times L$ diagonal matrix whose diagonal elements contain the $L$ largest eigenvalues and $\hat{\Lambda}_N$ stands for a diagonal matrix whose diagonal entries contain the $M-L$ smallest eigenvalues. $\hat{E}_S$ is the matrix composed of the eigenvectors corresponding to the $L$ largest eigenvalues of $\hat{R}_1$ while $\hat{E}_N$ represents the matrix including the remaining eigenvectors.

Then we construct the 1D MUSIC spectral peak search function as
\[ P_{\text{MUSIC}}(\beta) = \frac{1}{[Sb(\beta)]^H \hat{E}_N \hat{E}_N^H [Sb(\beta)]} \]  

Where \( b(\beta) = \left[ 1, e^{-j\omega_1\beta}, ..., e^{-j(M-1)\omega_1\beta} \right]^T \). To avoid the search the nulls of the denominator in the above spectrum are computed. Let \( z = e^{-j\omega_1\beta} \), \( p(z) = [1, z, ..., z^{M-1}]^T \) and \( p(z)^H = p(z^{−1})^T = [1, z^{-1}, ..., z^{-(M-1)}] \). Then the MUSIC null-spectrum function is constructed as follows:

\[ \hat{f}(z) = z^{N-1} [Sp(z)]^H \hat{E}_N \hat{E}_N^H [Sp(z)] \]  

Which represents the polynomial of degree 2(M-1). The roots of this polynomial appear in conjugate reciprocal pairs and, therefore, the initial estimates TOA corresponding to the antenna 1 can be estimated from the L largest roots \( z_1, z_2, ..., z_L \), which lay on or inside the unit circle, i.e.

\[ \hat{\beta}_{l,0} = -\frac{\text{angle}(\hat{z}_l)}{\Delta\omega}, \quad l = 1, 2, ..., L \]  

Now we make use of the relationship between the delays \( \hat{\beta}_l \) and \( \gamma_l \) for \( l = 1, 2, ..., L \) which is shown in equation (10) and get

\[ \beta_l - \frac{d}{c} \leq \gamma_l \leq \beta_l + \frac{d}{c} \]  

\( \gamma_l \) can be estimated as using the spatial spectrum of 1D-MUSIC shown as below:

\[ \hat{\gamma}_l = \text{arg max}_{\gamma \in [\beta_l - \frac{d}{c}, \beta_l + \frac{d}{c}]} \frac{1}{\hat{\beta}_l,0,0} a(\hat{\beta}_l,0,0) \]  

\[ l = 1, 2, ..., L \]  

We obtain the TOA estimates corresponding to antenna 2 by locally searching \( \gamma \) within \( \gamma \in [\beta_l - \frac{d}{c}, \beta_l + \frac{d}{c}] \). The TOAs \( \hat{\beta}_{l,0} \) and \( \hat{\gamma}_l \) in the spectral peaks are automatically paired. Then \( \alpha_l \) can be estimated using equation (20) by locally searching \( \beta_l \) within \( \beta \in [\beta_l - \Delta \tau, \beta_l + \Delta \tau] \) where \( \Delta \tau \) is a small value.

\[ \hat{\beta}_l = \text{arg max}_{\beta \in [\beta_l - \Delta \tau, \beta_l + \Delta \tau]} \frac{1}{\hat{\beta}_l,0,0} a(\beta, \hat{\gamma}_l) \]  

\[ l = 1, 2, ..., L \]  

Where \( \hat{\gamma}_l \) is the estimation of \( \gamma_l \).

The proposed Successive MUSIC algorithm can be summarized as follows:

- The covariance matrix of the received frequency-domain signal of antenna 1 is estimated and eigenvalue decomposition is performed to get the signal and noise subspace.
- The polynomial is constructed as shown in equation (16) and roots are obtained which are on or inside the unit circle and the initial estimation of TOAs \( \hat{\beta}_{l,0} \), \( l = 1, 2, ..., L \) can be estimated using equation (17).
- Matrix Z is constructed as shown in equation (12) and then eigenvalue decomposition is performed to get noise subspace \( \hat{U}_N \).
- TOA estimates of \( \hat{\gamma}_l \) are obtained through equation (19) while keeping \( \hat{\beta}_{l,0} \) fixed and the TOA estimates of \( \hat{\beta}_l \) are obtained through equation (20) while keeping \( \hat{\gamma}_l \) fixed.
- The DOA estimates \( \hat{\theta}_l \) are obtained using the relation shown in equation (11).

IV. SIMULATION RESULTS AND DISCUSSION

In this section, we present Monte Carlo simulations in order to evaluate the parameter estimation performance of the proposed algorithm. The signal to noise ratio (SNR) and the root mean squared error (RMSE) are defined as

\[ \text{SNR} = 10 \log \frac{\|x(t)\|^2_F}{\|n(t)\|^2_F} \]

and

\[ \text{RMSE} = \sum_{l=1}^{L} \frac{1}{M} \sum_{m=1}^{M} |\hat{\chi}_{l,m} - \chi_l|^2 \]

Where \( x(t) \) represents the received time-domain signal, \( n(t) \) represents the additive white Gaussian noise, \( M \) represents the Monte Carlo simulations, \( \hat{\chi}_{l,m} \) stands for the estimate of \( \chi_l \) of the \( m \)th Monte Carlo trial.

Figures 2 and 3 represents the UWB pulse waveform \( w(t) \) and the Transmitted signal \( s(t) \) which is generated using \( w(t) = \exp(-2\pi t^2/\Gamma^2) \) \( (1 - 4\pi t^2/\Gamma^2) \) with \( \Gamma = 0.25 \) ns as the shaping factor of the pulse and

\[ s(t) = \sum_{j=-\infty}^{\infty} \sum_{m=0}^{N_c-1} b_j c_m w(t - jT_s - mT_c) \]

With \( N_c=5 \), chip duration \( T_c=2 \) ns and symbol duration \( T_s=N_cT_c=10 \) ns.
Figures 4 and 5 are the estimated plots of TOAs and DOAs at [6ns, 15ns, 24ns, 35ns] and [-50°, -10°, 10°, 30°] respectively.

Figures 6 and 7 shows the plot of joint estimation of TOA and DOA for the proposed algorithm with SNR=0 dB over 50 Monte Carlo simulations. This determines that TOA and DOA can be estimated using the proposed algorithm and it works well with low SNR.
Figures 8 and 9 show that the TOA and DOA estimation performance with different $L$, which represents the number of multipath components. With increasing $L$, the interference among the components increases, which makes the algorithm lose its effectiveness in the low SNR region.

Figure 10 shows the comparison of the proposed Successive MUSIC algorithm with the existing algorithms. It displays that the proposed algorithm has better TOA estimation performance than PM algorithm, matrix pencil algorithm, Root MUSIC algorithm, ESPRIT algorithm, and estimation performance close to 2D-MUSIC algorithm.
Fig 10. TOA estimation comparison with various algorithms

Figure 11 shows the comparison of the proposed Successive MUSIC algorithm with the existing algorithms. It displays that the proposed algorithm has better DOA estimation performance than PM algorithm, matrix pencil algorithm, Root MUSIC algorithm, ESPRIT algorithm. From this figure we can see that the DOA estimation performance is better using 2D-MUSIC algorithm than the proposed Successive MUSIC algorithm.

Figures 12, 13 and 14 show the complexity comparison with different parameters. From these figures we find that the successive MUSIC algorithm has much lower computational complexity than the existing algorithms by varying the number of samples, number of clusters and number of multipaths. Here m and n refers to the number of steps in the 1D search range which is taken as m=2000 and n=100.
V. CONCLUSION

A Successive MUSIC algorithm for joint estimation of TOA and DOA in IR-UWB Positioning systems with a two-antenna receiver is presented in this paper. The proposed algorithm obtains the initial estimations of TOAs in the first antenna and then employs successive 1D search to achieve the estimation of 2D-TOA, and estimates the DOAs via the difference of TOAs between the two antennas. This Proposed algorithm can estimate the parameters jointly and can pair the parameters automatically. This Successive MUSIC algorithm provides a significant computational advantage over 2D-MUSIC algorithm and has better parameter estimation performance than PM algorithm, matrix pencil algorithm, Root-MUSIC algorithm and ESPRIT algorithm. Simulation results illustrate the accuracy and efficacy of the proposed algorithm in a variety of parameter and scenarios.

REFERENCES


BIOGRAPHY

