Tuning of a Feed forward Lag-Lead First-Order Compensator used with a Fractional Time Delay Double Integrating Process

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Abstract:

The problem of controlling an unstable delayed double integrating process with fractional delay using a feed forward first-order lag-lead compensator is studied. The effect of time delay of the process in a range between 0.1 and 0.9 seconds is considered. The compensator is tuned using MATLAB optimization toolbox with five forms of the objective function in terms of the error between the step time response of the closed-loop control system and the response steady-state value. Using the proposed compensator with the fractional delayed double integrating process indicates the robustness of the compensator in the time delay range used with superior time-based specifications compared with other technique based on PID controller.

Keywords — Delayed double integrating process with fractional delay, Feed forward lag-lead first-order compensator, compensator tuning, MATLAB optimization toolbox, Control system performance.

I. INTRODUCTION

Delayed double integrating processes are examples of unstable processes which require extensive efforts in proper selection of suitable controllers or compensators and also looking for proper tuning techniques to achieve stable control system and accepted performance both in the time and frequency domains. Researchers handled this type of processes over the last decades. Some of the efforts in this aspect are presented here.

De Paor (2002) developed a procedure for the control of integrating and unstable processes with time delay. He presented examples including simple integrating and double integrating delayed processes [1]. Lin, He, Gu and Zhang (2004) proposed two control schemes to control dynamic plants with time delay and double integrators. They analysed the closed-loop robust stability and corresponding sufficient and necessary conditions in the presence of plant uncertainty [2]. Shamsuzzoha and Lee (2008) proposed a simple IMC-PID controller design technique for the first-order delay integrating process, delayed integrating process and double integrating process. They focused on the disturbance rejection and used a 2DOF control structure to eliminate the set point response overshoot [3].

Wang and Zhang (2009) discussed the problem of determining the parameters sets of stabilizing PID controllers for first-order plus double integrating processes with time delay. They presented case studies to illustrate the design procedure showing the shapes of the stabilizing regions [4]. Zhou, Duan and Lin (2010) studied the dynamic problem of global stabilizing a double integrator system subject to actuator saturation and input delay. They illustrated the effectiveness of their approach through an example [5]. Vasickaninova and Bakosova (2011) studied the tuning of neuro-fuzzy controllers for integrating plant and for integrating plants with time delay. They compared with classical PID control [6]. Skogestad and Grimholt (2012) studied the optimality of the SIMC PI rules using IAE performance. They presented applications for first-order, second-order integrating with lag, integrating and double integrating delayed processes [7].

Thirunavukkarasu, Zyla, George and Priya (2013) discussed the tuning of a PID controller with time delay for double integrator systems for particular stability margins. They considered as a case study a double integrating process of 0.7 gain and 0.1 s time delay [8]. Hassaan (2014) used a first-order
lag-lead feedforward compensator to control a simple pole plus an undelayed double integrator process. He used an ISE objective function incorporating the errors between the closed-loop system gain and phase margins and a desired levels of the two stability parameters. He compared his results with other published work [9]. Anil and Sree (2015) designed a PID controller for various forms of integrating systems with time delay using the direct synthesis method. They adjusted a tuning parameter to achieve a desired robustness. They provided tuning rules in terms of process parameters for various forms of integrating systems [10]. Hassaan (2015) used a first-order lag-lead compensator to control a very slow second-order process of 150 s settling time. He tuned the compensator using an ISE objective function and could reduce the settling time to only 0.666 s [11]. Hassaan (2015) studied the control of a delayed double integrating process of time delay from 1 to 7.4 s using a feedforward first-order compensator. He used different objective functions and compared his results with other research works [12].

II. PROCESS

The controlled process is delayed double integrating process having the transfer function, \( G_p(s) \):\
\[
G_p(s) = \left(\frac{K_p}{s^2}\right)\exp(-T_ds) \tag{1}
\]
Where \( K_p \) is the process gain and \( T_d \) is its time delay.

It is dealt with the exponential term in Eq.1 through the first-order Taylor series as [7]:\
\[
\exp(-T_ds) \approx 1 - T_ds \tag{2}
\]

Combining Eqs.1 and 2 gives the process transfer function as:
\[
G_p(s) = \left(-\frac{K_p}{T_d}s + K_p\right) / s^2 \tag{3}
\]
The unit step response of the process using Eq.3 is shown in Fig.1.

Fig.1 Step response of the double integrating process.

It is clear from Fig.1 that the double integrating process is an unstable 1. The compensator has to generate an stable feedback control system and also to achieve an accepted performance through tuning the compensator.

III. COMPENSATOR

The compensator used is a feedforward first-order compensator having the transfer function, \( G_c(s) \) [9, 11, 13]:
\[
G_c(s) = \frac{K_c(1 + T_zs)}{1 + T_ps} \tag{4}
\]

Where \( K_c \) is the compensator gain, \( T_z \) and \( T_p \) are two time constants of the compensator.

IV. CONTROL SYSTEMTRANSFER FUNCTIONS

For a unit feedback control system, the open-loop transfer function of the control system incorporating the feedforward first-order compensator and the delayed double integrating process, \( G(s)H(s) \) is given by:
\[
G(s)H(s) = \frac{b_0s^2 + b_1s + b_2}{T_ps^3 + s^2} \tag{5}
\]

Where:
\[
b_0 = -K_cK_pT_zT_d \\
b_1 = K_cK_pT_z - K_cK_pT_d \\
b_2 = K_cK_p
\]

The closed-loop transfer function of the control system, \( M(s) \) is given for a unit feedback elements by:
\[
M(s) = \frac{(b_0s^2 + b_1s + b_2)[T_ps^3 + (1 + b_0)s^2 + b_1s + b_2]}{[T_ps^3 + (1 + b_0)s^2 + b_1s + b_2]} \tag{6}
\]

V. COMPENSATOR TUNING AND SYSTEM TIME RESPONSE

The compensator is relatively simple having only three parameters: \( K_c, T_z \) and \( T_p \). The compensator parameters are tuned as follows:

- The compensator gain is selected to achieve a zero steady-state error. Using Eq.6, this condition reveals a unit compensator gain.
- The optimization toolbox of MATLAB is used to assign the other two parameters of the compensator (\( T_z \) and \( T_p \)) [14].
- The MATLAB command 'fminunc' is used [14].

- A number of objective functions based on the error between the step time response of the control system and its steady-state response are selected to tune the compensators. They are ITAE, ISE, IAE, ITSE and ISTSE [15-18].

- The tuning procedure is applied for a specific time delay of the double integrating process in the range $0.1 \leq T_d \leq 0.9$ s.

- The step response of the closed-loop control system is plotted using the command 'step' of MATLAB [19].

- The time-based specifications of the control system are extracted using the MATLAB command 'stepinfo' [19].

- The frequency-based specifications of the control system are extracted using the MATLAB command 'margin' [19].

A sample of the tuning results is shown in Table 1 for a unit time delay of the double integrating process and a unit gain.

<table>
<thead>
<tr>
<th>Objective Function</th>
<th>$T_z$</th>
<th>$T_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITAE</td>
<td>1.6785</td>
<td>0.2908</td>
</tr>
<tr>
<td>ISE</td>
<td>3.0452</td>
<td>0.6005</td>
</tr>
<tr>
<td>IAE</td>
<td>2.9144</td>
<td>0.5080</td>
</tr>
<tr>
<td>ITSE</td>
<td>2.0557</td>
<td>0.4114</td>
</tr>
<tr>
<td>ISTSE</td>
<td>1.7126</td>
<td>0.3041</td>
</tr>
</tbody>
</table>

The time response of the control system for a unit step input is shown in Figs.2 for time delay of 0.2 s.

Varying the type of the optimization objective function has a remarkable effect the time response of the control system. The best of them for fractional time delay of the double integrating processes is the ITAE and ISTSE objective functions.

The effect of time delay of the time response of the closed-loop control using ITAE in tuning the feedforward first-order compensator is shown in Fig.3.

The effect of the time delay of the double integrating process on some of the time-based specifications of the control system is shown in Fig.4 using the ITAE objective function.
The settling time decreases as the time delay increases from 0.1 to 0.9 s. The maximum percentage overshoot changes between 0.049 and 0.864 % during the time delay range between 0.1 and 0.9 s with 0.25 standard deviation. The mean maximum percentage overshoot is 0.386 %.

VI. COMPARISON WITH OTHER RESEARCH WORK

The unit time response of the control systems as presented in the present work using a first-order feedforward compensator is compared with the work of Skogestad and Grimbolt [7] using tuned PID control to control the delayed double integrating process. The comparison is presented graphically in Fig.5 for an 0.5 s time delay.

The present work gives outstanding time response compared with the other techniques. The compared work is for a process having unit gain and unit time delay.

VII. CONCLUSION

- The dynamic problem of tuning a feedforward lag-lead compensator for use with a fractional delayed double integrating process was investigated.
- The effect of the process time delay on the tuning results and the time response of the control system was investigated.
- Five objective function forms were applied to tune the compensator.
- It has been shown that choosing a suitable objective function had an important role in tuning the compensator and producing good performance for the closed-loop control system.
- ITAE and ISTSE objective functions were the best objective function used in tuning the feedforward first order compensators for double integrating processes with fractional delay time.
- The proposed compensator succeeded to produce a time response to a step reference input having a maximum percentage overshoot as low as 0.049 % at an 0.9 s process time delay.
- Corresponding to the time delay range covered in the analysis (0.1 to 0.9 s), the maximum percentage overshoot did not exceed 0.864 %.
- The settling time of the control system step response did not exceed 4.53 seconds during the covered time delay range.
- The feedforward first-order lag-lead compensator was robust against the change in the fractional process time delay in the range between 0.1 and 0.9 seconds.
- Although the studied delayed double integrating process was an unstable one, the proposed compensator has given superior results when compared with other research works based on using PID controllers.
- For a process of unit gain and 0.5 s time delay, the maximum percentage overshoot was 0.149 % for the present work compared with 1.114 % for the Skogestad and Grimholt work.
- The settling time was 3.045 s for the present work compared with 7.5 s for the Skogestad and Grimholt work.

REFERENCES

2. T. Lin, X. He, D. Gu and W. Zhang, “Analytical decoupling control for dynamic plants with time delay and

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