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Analysis of C-F Short Cylindrical Shells under Internal Hydrostatic Pressure using Polynomial Series Shape Function.

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Abstract:

The analysis of short cylindrical shells with one end clamped and the other end free (C-F) under internal hydrostatic pressure was presented. The study was carried out through a theoretical formulation based on polynomial series and application of conservation of work principle. By satisfying the boundary conditions of the C-F short cylindrical shell, a particular shape function was obtained which was substituted into the total potential energy functional of the conservation of work principle and minimized to obtain the unknown coefficient. Aspect ratios considered were in the range 1< L/r <4, stresses and strains at various points of the C-F short cylindrical shell were determined.

The most critical case 1, deflection, rotation, bending moment and shear force were $6.62898 * 10^{-4} meters, -8.28623 * 10^{-4} radians, -119.8900 KNm, \ -359.06997 KN$ respectively.

It was observed that as the aspect ratio increases, the stresses and strains decreases in magnitude.

Keywords — **Boundary conditions, C-F short cylindrical shell, Conservation of work principle, Internal hydrostatic pressure, Polynomial series shape function.**

I. INTRODUCTION

Large cylindrical shell tanks are widely used in the construction of strategic water or oil reservoir all over the world (Yukun et al, 2013) [1]. The larger the tanks are built, the higher the hydrostatic pressure due to the liquid it contained and thus the higher the risk of failure. Failure of this structure is catastrophic as it involves huge economic loss, risk to life and unimaginable environmental degradation. Therefore, more attention should be given to a thorough and comprehensive structural analysis of such structures, which will lead to safe design and construction. C-F short cylindrical shell is a typical example of these large cylindrical shell reservoirs used as a hydraulic structure for water storage.

Osadebe and Adamou [2] studied Static Analysis of Cylindrical shell under hydrostatic and Ring forces using initial value method. Ventsel and Krauthammer [3] studied short cylindrical shell using the Krylov's function but did not consider axisymmetrically loaded shell with internal hydrostatic pressure. Ezeh et al [4] studied Static Analysis of C-S short cylindrical shell under internal liquid pressure but did not consider the case of C-F boundary condition. Agbo et al, 2015 [5] studied the analysis of C-C short cylindrical shell under internal liquid pressure but did not consider the case of C-F boundary condition.

Thin shells as structural elements occupy a leadership position in engineering and especially in civil engineering, since they can be used in the construction of large liquid storage structures, large span roofs, domes, folded plates and so on.

This work is concerned with the analysis of C-F short cylindrical shell subjected to axisymmetric internal hydrostatic pressure.

The objective of this study is to establish an analysis of C-F short cylindrical shell that can be utilized in the design of cylindrical shell reservoir tank of one end fixed and the other end free under hydrostatic pressure. To show the deflections and stress distributions on the C-F shell reservoir under internal hydrostatic pressure. The locations and magnitudes of the maximum values of deflections and stresses were indicated and are useful for design.

II. DIFFERENTIAL EQUATION OF CYLINDRICAL SHELL.

A C- F short cylindrical shell with dimensions L, t and r, as shown in figure 1 which is subjected to axisymmetric internal hydrostatic pressure.

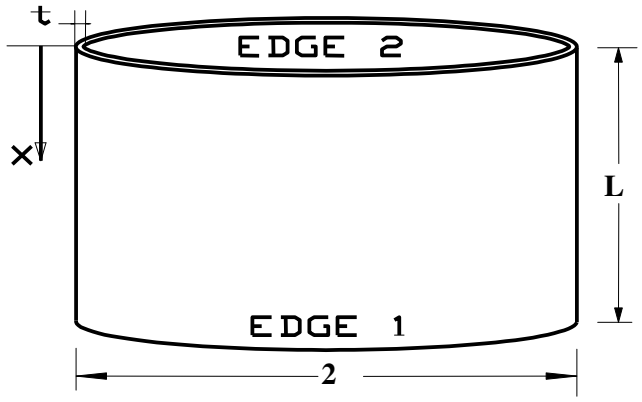

Figure 1: A typical C-F (clamped at one end and free at the other end) short cylindrical shell tank showing the dimensions.

According to Ezeh et al, 2014 [4], the condition for shortness for an unstiffened cylindrical shell is L/r < 5, where L/r is the aspect ratio.

The governing equation of a cylindrical shell according to the semi-moment theory as used by Timoshenko et al [6]; Ventsel and Krauthammer [3] is as stated in equation (1).

$$\frac{d^4w}{dx^4} + 4\beta^4 w = \frac{\gamma}{D}\mathrm{x} \qquad (1)$$

Where

$$\beta^4 = \frac{3(1-v^2)}{r^2t^2} \qquad (2)$$

γ = Unite weight of the liquid.

Equation (1) is due to Pasternak [7] and is only applicable to cylindrical shell subject to axisymmetric loading.

Ezeh et al, 2014 [4] gave the general polynomial series shape function for short cylindrical shell as:

$$\mathrm{w} = k_0 + k_1R + k_2R^2 + k_3R^3 + kR^5 \qquad (3)$$

Where,

$$k_0 = a_0\mathrm{c};\ k_1 = a_1\mathrm{c}\,;\ k_2 = a_2\mathrm{c};\ k_3 = a_3\mathrm{c};\ k_5 = \Phi\mathrm{b} \qquad (4)$$

III. THE CONSERVATION OF WORK PRINCIPLE.

The conservation of work principle used in the formulation of the solution for the C-F short cylindrical shell was obtained from Ibearugbulem et al [8].

$$\int_0^L \left(\frac{d^4w}{dR^4}\right)\mathrm{wdR} + \lambda\int_0^L w^2\,dR - \int_0^L \Phi\, wRdR = 0 \quad (5)$$

IV. SHAPE FUNCTION FOR C-F SHORT CYLINDRICAL SHELL.

The C-F short cylindrical shell has the following boundary conditions.

$$M_0 = \frac{d^2w}{dR^2}(R=0) = 0 \qquad (6)$$

$$Q_0 = \frac{d^3w}{dR^3}(R=0) = 0 \qquad (7)$$

$$w_{(1)} = w(R=1) = 0 \qquad (8)$$

$$\theta_{(1)} = \frac{\mathrm{dw}}{\mathrm{dR}}(R=1) = 0 \qquad (9)$$

Substituting the boundary conditions in equations (6),(7), (8) and (9) into equation (3) and solving the resulting equations gave

$$w = A(4 - 5R + R^5) \ = Af_1 \qquad (10)$$

Equation (10) is the shape function for C-F short cylindrical shell under internal hydrostatic pressure.

V. CONSERVATION OF WORK PRINCIPLE SOLUTION FOR C-F SHORT CYLINDRICAL SHELL.

The shape function for C –F short cylindrical shell is given by equation (10) as:

$$w = A(4 - 5R + R^5) \qquad (10)$$

Using the conservation of work principle solution as stated in equation (5) as follows:

$$\int_0^L \left(\frac{d^4w}{dR^4}\right)\mathrm{wdR} + \lambda\int_0^L w^2\,dR - \int_0^L \Phi\, wRdR = 0 \quad (5)$$

Using equation (5), the following values were obtained:

$$\frac{\mathrm{d^4w}}{\mathrm{dR^4}}.\mathrm{w} = 120A^2(4R - 5R^2 + R^6) \qquad (11)$$

$$w^2 = A^2(16 - 40R + 8R^5 + 25R^2 - 10R^6 + R^{10}) \qquad (12)$$

$$wR = A(4R - 5R^2 + R^6) \qquad (13)$$

Integrating equations (11), (12) and (13) with respect to R in a close domain gave the following:

$$\int_0^1 \left(\frac{d^4w}{dR^4}\right) w dR = 120A^2\left(\frac{4}{2} - \frac{5}{3} + \frac{1}{7}\right)$$

$$= 57.1428571A^2 \qquad (14)$$

$$\int_0^1 (w)^2 dR = A^2\left(16 - \frac{40}{2} + \frac{8}{6} + \frac{25}{3} - \frac{10}{7} + \frac{1}{11}\right)$$

$$= 4.329004A^2 \qquad (15)$$

$$\int_0^1 (wR) dR = A\left(\frac{4}{2} - \frac{5}{3} + \frac{1}{7}\right)$$

$$= 0.4761905A \qquad (16)$$

Substituting equations (14), (15) and (16) into equation (10) and simplifying further gave:

$$57.1428571A^2 + 4.329004A^2\lambda - 0.4761905A\Phi = 0 \qquad (17)$$

Making A the subject of equation (17) gave:

$$A = \frac{0.4761905\ \Phi}{57.1428571 + 4.329004\lambda} \qquad (18)$$

Knowing that:

$$\Phi = \frac{\gamma L^5}{D}$$

$$\lambda = \frac{12L^4(1 - v^2)}{r^2t^2}$$

Substituting for Φ and λ, and simplifying further, we have:

$$A = \frac{0.11\ \gamma L^5 r^2}{13.20\ r^2 D + EtL^4} \qquad (19)$$

Substituting equation (19) into equation (10) gave:

$$w = \frac{0.11\ \gamma L^5 r^2}{13.20\ r^2 D + EtL^4}(4 - 5R + R^5) \qquad (20)$$

Equation (20) is the shape function for a C-F short cylindrical shell using polynomial series in the conservation of work principle.

Deflection (w) is given as:

$$w = \frac{0.11\ \gamma L^5 r^2}{13.20\ r^2 D + EtL^4}(4 - 5R + R^5) \qquad (21)$$

Slope (rotation) (θ).Differentiating equation (21) with respect to R gave the slope.

$$\theta = \frac{dw}{dR} = \frac{0.11\ \gamma L^5 r^2}{13.20\ r^2 D + EtL^4}(5R^4 - 5) \qquad (22)$$

Bending moment (M).

Differentiating equation (22) with respect to R and multiplying with the negative value of the flexural rigidity D gave the bending moment.

$$M = -D\left(\frac{d^2w}{dR^2}\right)$$

$$= -D\left[\frac{0.11\ \gamma L^5 r^2}{13.20\ r^2 D + EtL^4}\right](20R^3) \qquad (23)$$

Shear force (Q).

Differentiating equation (23) with respect to R gave the shear force.

$$Q = -D\left(\frac{d^3w}{dR^3}\right)$$

$$= -D\left[\frac{0.11\ \gamma L^5 r^2}{13.20\ r^2 D + EtL^4}\right](60R^2) \qquad (24)$$

VI. NUMERICAL STUDIES

The stresses and strains at various points of C-F short cylindrical shells were determined for values of aspect ratios ranging from 1 to 4. The equations of the deformations and stresses of C-F short cylindrical shells are presented. The numerical values of the following parameters E, D, L, t, r and γ are substituted accordingly into the formulated solutions.

VII. RESULTS AND DISCUSSION

The graphs of each of the deflections, slopes, bending moments and shear forces for C-F short cylindrical shell of cases 1 to 4 were plotted together against the height of the tank and presented as follows:

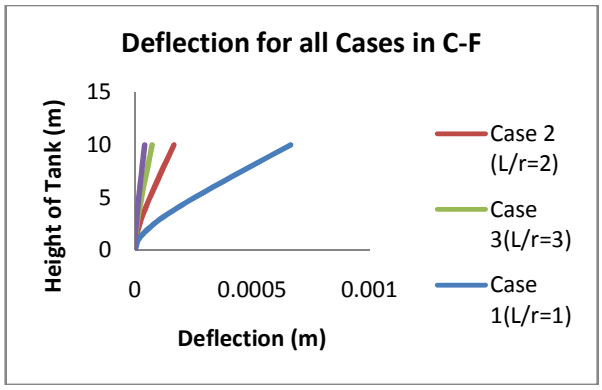

Figure 2: Deflection curves for C-F short cylindrical shells of aspect ratios 1-4.

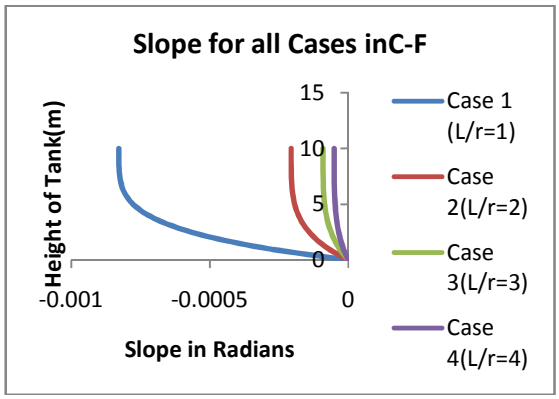

Figure 3: Rotation curves for C-F short cylindrical shells of aspect ratios 1-4.

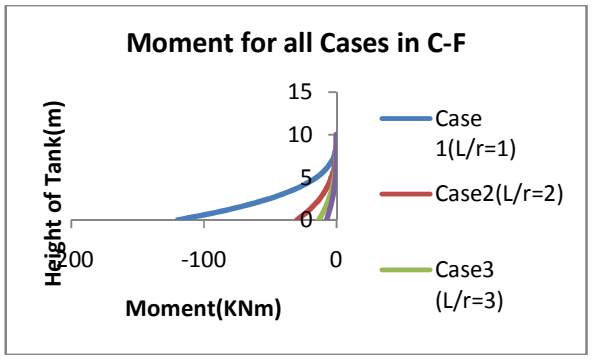

Figure 4: Bending moment diagrams for C-F short cylindrical shells of aspect ratios 1-4.

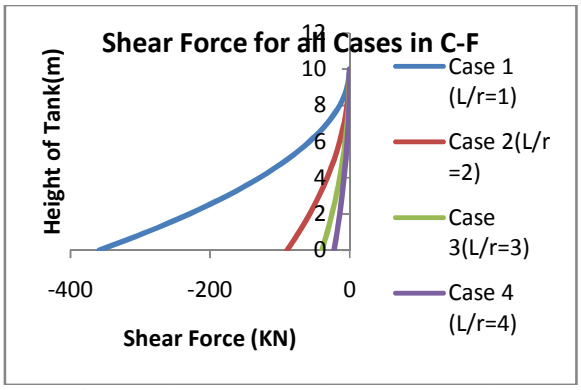

Figure 5: Shear force diagrams for C-F short cylindrical shells of aspect ratios 1-4.

Table 1: Maximum values of deflections, rotations, bending moments and shear forces for C-F short cylindrical shells.

Cases	Maximum Deflection (m)	Maximum Rotation (radians)	Maximum Bending moment (KNm)	Maximum Shear force (KN)
1	$6.62898 * 10^{-4}$	$-8.28623 * 10^{-4}$	-119.8900	-359.06997
2	$1.65816 * 10^{-4}$	$-2.0727 * 10^{-4}$	-29.9390	-89.81687
3	$7.35361 * 10^{-5}$	$-9.19451 * 10^{-5}$	-13.2810	-39.84287
4	$4.14596 * 10^{-5}$	$-5.18246 * 10^{-5}$	-7.48576	-22.45731

Deflection:

From the graph shown in Figure 2, it was observed that the deflection curve has a parabolic-like shape with the maximum values at the top of the shell, which is the free edge. The behaviour of the graph showed that as the aspect ratio increases, from 1 to 4, the deflection decreased.

Rotation:

From the graphs shown in Figure 3, it was observed that the slope (rotation) has the maximum values at the top of the shell which is the free edge. The behavior of the graph showed that as the aspect ratio increases from 1 to 4, the rotation decreased.

Bending moment:

From the graphs shown in Figure 4, it was observed that the bending moment described a parabolic-like curve due to the cantilever action with the maximum values at the base of the shell which is the clamped edge and zero at the free edge. The behaviour of the graph showed that as the aspect ratio increased from 1 to 4, the bending moment decreases.

Shear force:

From the graphs of Figure 5, it was observed that the shear force varied along the height of the shell with the maximum values at the clamped base of the shell and zero at the free edge. The behaviour of the graph showed that as the aspect ratio increases from 1 to 4, the shear force decreased.

VIII. CONCLUSION

Using the polynomial series in the Conservation of work principle is more convenient for analyzing C-F short cylindrical shells than the use of krylov's function. Knowledge of the point of maximum stresses along the height of the shell help for adequate reinforcement to be provided at the appropriate point. In the case of stiffening the shell with rings, this guides in the position of the rings for optimal design.

It is therefore recommended that this approach could be easily applied in solving C-F short cylindrical shell problems during the design of large cylindrical shell water reservoir.

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