

# Creation of Oscillation Death from Amplitude Death in Non-Identical Dual Channel Coupled Chua Circuit

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## Abstract:

The important phenomenon of creation of oscillation death (OD) from amplitude death (AD) state in coupled Chua's circuit has been explored here. The mean field diffusively (MFD) coupled Chua's circuits are when additionally coupled through simple diffusive coupling through other channel, it exhibits the OD-AD transition phenomena. The effect of design parameters and coupling parameters are investigated in details. These are examined through numerical simulations for both identical and non-identical systems. The stability of trivial steady states is predicted theoretically. A hardware experiment is also performed to observe the OD in the system. The present study includes a new dimension to versatility of Chua's circuit in the field of nonlinear dynamics.

**Keywords** — Transition, Oscillation death, Amplitude death, Dual channel coupling.

## I. INTRODUCTION

Recently, the study on the coupled system gets great importance. A wide variety of interesting complex collective phenomena like synchronization, dynamical hysteresis, phase flip, oscillation quenching etc. having potential application are found in different coupled systems [1]-[7]. Oscillation quenching is a phenomenon where the suppression of oscillatory behavior occurs through the interaction of coupled system. We found two types of oscillation quenching; Amplitude death (AD) and Oscillation death (OD) [8]. In AD the coupled system converges to homogeneous steady state (HSS) whereas in OD the system converges to heterogeneous i.e. inhomogeneous steady state (IHSS). Oscillation quenching is mainly important to control and stabilize the system viz. in laser [9], neuronal systems [10], [11], etc. In spite of this, it has found applications in diverse field [7], [12], [13].

The simultaneous occurrence of AD and OD and the transition from AD to OD under diffusive coupling in coupled Stuart-Landau oscillators had been reported by Koseska et al. in [12]. After then several works have been reported showing such transition in different coupled system like; mean

field diffusive (MFD) coupled system [14], [15], time-delayed system [16], dynamic coupled system [17], conjugate coupled system [18], [19], diffusive and repulsive coupled system [20], [21] etc.

Most of the cases the quenching phenomena and their transitions had been assumed the case of one-channel coupling. But, the interactions between coupled oscillators can be more complicated [22]. The ability of synchronization of the complex network has been highly affected by dual channel coupling [23]. It is also reported that, two [24] or multi channel delay [25] have been introduced to stabilize the chaotic systems. The effect of delay and the multi-channel coupling on the region of OD in coupled identical oscillators had been reported in [26].

But, the effect of more than one channel coupling on the dynamics of coupled system are still not properly ascertain. In [27] the effects of identical dual-channel coupling on the AD and OD region, and their transition in coupled non-identical oscillators were explored. But in this paper, we examined the AD-OD transition scenario in Chua circuit using non-identical dual channel coupling between them. Such type of coupling is also known as direct-indirect coupling. Chua's circuit is one of the most simple and efficient circuit that exhibits

wide variety of nonlinear dynamics within it. It can also be easily implemented in experiments [28]-[30]. So far literature is concern, a good amount of works have already been done on the coupled Chua's circuit using different coupling schemes [19], [31-35]. But in all cases the coupled system has been under single channel coupling and converges to a common steady state showing AD in them. Sharma et al. already reported AD with mean field diffusion in [34], [35]. We explore the dynamics of such MFD coupled Chua's circuit when an additional diffusive coupling has been employed through another channel. It is well known that Chua's circuit is a third order autonomous system and its dynamics is described by three state variables [30]. In our work, we coupled one of the state variables of Chua's circuits through MFD coupling and another state variable through a simple variable resistor (i.e. direct diffusively coupled). Interestingly, in such condition the system shows the transition between AD and OD depending upon the coupling strength. The OD is appeared through symmetry breaking. Such dynamics are described through numerical simulation and also supported by a prototype hardware experiment. The stability of steady states is derived theoretically using Routh-Hurwitz technique.

The paper has been organized as follows. The equations describing the system dynamics for such modified MFD coupled Chua's circuit has been formulated in section 2. The derivations of the steady state points and their stability have been examined analytically in section 3. The occurrences of AD, OD and their transition in the system have been discussed in section 4 through numerical simulations. The effect of different coupling and design parameters on the quenching dynamics of the system have also been discussed here in details. In establishment of such dynamics we perform an experiment on a proto type hardware circuit using off-the-self ICs. The details of the experiments and the experimental results are described in section 5. Some concluding remarks are given in the Section 6.

## II. SYSTEM EQUATIONS FOR MODIFIED COUPLED CHUA'S CIRCUITS

The complete hardware circuit diagram of coupled Chua's circuits under non-identical dual channel coupling is shown in Fig. 1. First, we connect the branch points having voltage  $v_{12}$  and  $v_{22}$  through MFD coupling. Using the Kirchhoff's current law we derive the following equations for MFD coupled Chua's circuits are [30],

$$C_{11} \frac{dv_{11}}{dt} = \frac{(v_{12}-v_{11})}{r_{11}} - f(v_{11}) \quad (1a)$$

$$C_{12} \frac{dv_{12}}{dt} = \frac{(v_{11}-v_{12})}{r_{11}} + i_{1L} + \left(\frac{1}{R_6}\right) \left(\frac{R_3}{R_1}(v_{12} + v_{22}) - (v_{12})\right) \quad (1b)$$

$$L_1 \frac{di_{1L}}{dt} = -v_{12} \quad (1c)$$

$$C_{21} \frac{dv_{21}}{dt} = \frac{(v_{22}-v_{21})}{r_{21}} - f(v_{21}) \quad (1d)$$

$$C_{22} \frac{dv_{22}}{dt} = \frac{(v_{21}-v_{22})}{r_{21}} + i_{2L} + \left(\frac{1}{R_6}\right) \left(\frac{R_3}{R_1}(v_{12} + v_{22}) - (v_{22})\right) \quad (1e)$$

$$L_2 \frac{di_{2L}}{dt} = -v_{22} \quad (1f)$$

Here, the first digit of subscript (i.e. 1 or 2) is used to represent first or second Chua's circuit, respectively.  $f(v_{11})$  and  $f(v_{21})$  in equation (1a) and (1d) represent the response of the nonlinear resistor of two Chua circuits respectively and it can be mathematical represented as follows,

$$f(v_{11,21}) = m_0 v_{11,21} + \left(\frac{1}{2}\right) (m_1 - m_0) [ |v_{11,21} + B_p| - |v_{11,21} - B_p| ] \quad (2)$$

Here,  $m_0$  ( =  $-\left(\frac{R_2}{R_1 R_3}\right) + \left(\frac{1}{R_4}\right)$  ) and  $m_1$  ( =  $-\left(\frac{R_2}{R_1 R_3}\right) - \left(\frac{R_5}{R_4 R_6}\right)$  ) are the inner and outer slopes of the typical characteristic curve of the Chua's nonlinear resistor.  $B_p$  is the breakpoint of two slopes (which are considered identical for both circuits). Now we connect the branch points having the voltage  $v_{11}$  and  $v_{21}$  through a simple resistor  $R_7$  (Fig.1). Thus, the two Chua's circuits are direct diffusively coupled through these state variables. Then the above equation set are modified as given below,

$$C_{11} \frac{dv_{11}}{dt} = \frac{(v_{12}-v_{11})}{r_{11}} - f(v_{11}) + \frac{(v_{11}-v_{21})}{R_7} \quad (3a)$$

$$C_{12} \frac{dv_{12}}{dt} = \frac{(v_{11}-v_{12})}{r_{11}} + i_{1L} + \left(\frac{1}{R_6}\right) \left(\frac{R_3}{R_1} (v_{12} + v_{22}) - (v_{12})\right) \quad (3b)$$

$$L_1 \frac{di_{1L}}{dt} = -v_{12} \quad (3c)$$

$$C_{21} \frac{dv_{21}}{dt} = \frac{(v_{22}-v_{21})}{r_{21}} - f(v_{21}) - \frac{(v_{11}-v_{21})}{R_7} \quad (3d)$$

$$C_{22} \frac{dv_{22}}{dt} = \frac{(v_{21}-v_{22})}{r_{21}} + i_{2L} + \left(\frac{1}{R_6}\right) \left(\frac{R_3}{R_1} (v_{12} + v_{22}) - (v_{22})\right) \quad (3e)$$

$$L_2 \frac{di_{2L}}{dt} = -v_{22} \quad (3f)$$

Considering  $\tau = \frac{t}{C_{12}r_{11}}$ , a dimensionless quantity, we normalized the above equations and derive six normalized state equations describing the dynamics of such dual channel coupled Chua circuit as,

$$\frac{dx_1}{d\tau} = \alpha_1 \{ (y_1 - x_1 - f(x_1)) + g_1(x_1 - x_2) \} \quad (4a)$$

$$\frac{dy_1}{d\tau} = x_1 - y_1 + z_1 + d_1 \left( m \left( \frac{y_1+y_2}{2} \right) - y_1 \right) \quad (4b)$$

$$\frac{dz_1}{d\tau} = -\beta_1 y_1 \quad (4c)$$

$$\frac{dx_2}{d\tau} = \alpha_2 \{ (y_2 - x_2 - f(x_2)) - g_2(x_1 - x_2) \} \quad (4d)$$

$$\frac{dy_2}{d\tau} = \gamma \{ (x_2 - y_2 + z_2) + d_2 \left( m \left( \frac{y_1+y_2}{2} \right) - y_2 \right) \} \quad (4e)$$

$$\frac{dz_2}{d\tau} = -\beta_2 y_2 \quad (4f)$$

Here,  $x_{1,2} = \frac{v_{11,21}}{B_p}$ ,  $y_{1,2} = \frac{v_{12,22}}{B_p}$ ,  $z_{1,2} = \frac{r_{11,21}i_{L1,2}}{B_p}$ ,  $\alpha_1 = \left(\frac{C_{12}}{C_{11}}\right)$ ,  $\alpha_2 = \left(\frac{C_{12}r_{11}}{C_{21}r_{21}}\right)$ ,  $\gamma = \left(\frac{C_{12}r_{11}}{C_{22}r_{21}}\right)$ ,  $\beta_1 = \left(\frac{C_{12}r_{11}^2}{L_1}\right)$ ,  $\beta_2 = \left(\frac{C_{12}r_{11}r_{21}}{L_2}\right)$ ,  $f(x_1) = r_{11}f(v_{11})$ ,  $f(x_2) = r_{21}f(v_{21})$ . The factors  $d$  [ $d_1 = \left(\frac{r_{11}}{R_6}\right)$  and  $d_2 = \left(\frac{r_{21}}{R_6}\right)$ ],  $m = \left(\frac{2R_3}{R_1}\right)$  and  $g$  [ $g_1 = \frac{r_{11}}{R_7}$  and  $g_2 = \frac{r_{21}}{R_7}$ ] signifies the diffusive strength, mean field strength and strength of direct coupling of the coupled system, respectively.

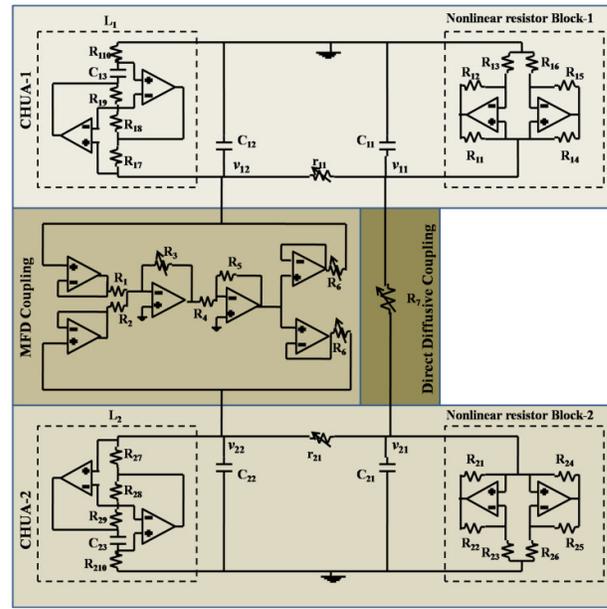


Fig. 1 The hardware circuit diagram for experiment of modified MFD coupled Chua's circuits

### III. STABILITY ANALYSIS

In this section at first we calculate the different steady states in quenched condition of the coupled system and then we assess analytically the limit of those steady states (oscillation quenched) using Routh-Hurwitz technique.

In case of AD, we have  $x_1^* = x_2^*$ ,  $y_1^* = y_2^*$ ,  $z_1^* = z_2^*$ . Putting these conditions and equating the left hand side of equation (4) equal to zero, we can see that the system has the following steady state values,

- (i)  $(x_1^*, 0, -x_1^*, x_1^*, 0, -x_1^*)$ , for,  $x_{1,2} < -1$ , (ii)  $(0, 0, 0, 0, 0, 0)$ , for,  $|x_{1,2}| < 1$  and (iii)  $(-x_1^*, 0, x_1^*, -x_1^*, 0, x_1^*)$ , for,  $x_{1,2} > 1$ , where,  $x_1^* = \left[ \frac{(m_1 - m_0)r_{11}}{(1 + m_0)r_{11}} \right]$ . These are the trivial steady states of the system.

In case of OD, we have  $x_1^* = -x_2^*$ ,  $y_1^* = -y_2^*$ ,  $z_1^* = -z_2^*$ . Putting these conditions in equation (4), we can get the following steady state values,

- (i)  $(x_1^*, 0, -x_1^*, x_1^*, 0, -x_1^*)$ , for,  $x_{1,2} < -1$ , (ii)  $(0, 0, 0, 0, 0, 0)$ , for,  $|x_{1,2}| < 1$  and (iii)  $(-x_1^*, 0, x_1^*, -x_1^*, 0, x_1^*)$ , for,  $x_{1,2} > 1$ , where,  $x_1^* = \left[ \frac{(m_1 - m_0)r_{11}}{(1 + m_0)r_{11} \pm 2g_{1,2}} \right]$ . These are non-trivial steady states and depend on the strength of direct coupling. Note

that, the OD states are born through symmetry breaking if we have  $\mathbf{g}_1 = \mathbf{g}_2$ . Otherwise, the two states of OD will not be symmetric. This can be done by taking  $\mathbf{r}_{11}$  and  $\mathbf{r}_{21}$  different.

Next, we find the stability of the trivial steady state values. Here, the linear transformation Jacobian matrix of the system is constructed at the steady state points with  $x_{1,2} > 1$  using the relation  $\mathbf{x}_1^* = \mathbf{x}_2^*$ ,  $\mathbf{y}_1^* = \mathbf{y}_2^*$ ,  $\mathbf{z}_1^* = \mathbf{z}_2^*$ . The characteristic equation is a cubic equation of the eigen value  $\lambda$  and it is as given:

$$\lambda^3_{1,2,3} + (\mathbf{k}_1 + \mathbf{dP}_2)\lambda^2_{1,2,3} + (\mathbf{k}_2 + \mathbf{dP}_1\mathbf{P}_2\alpha_1)\lambda_{1,2,3} + \mathbf{P}_1\alpha_1\beta_1 = 0 \quad (5)$$

Where,  $\mathbf{P}_1 = (\mathbf{1} + \mathbf{m}_0\mathbf{r}_{11})$ ,  $\mathbf{P}_2 = (\mathbf{1} - \mathbf{m})$ ,  $\mathbf{k}_1 = (\mathbf{1} + \mathbf{P}_1\alpha_1)$  and  $\mathbf{k}_2 = (\beta_1 + \mathbf{m}_0\mathbf{r}_{11}\alpha_1)$ . Applying the Routh-Hurwitz array technique to the equation (5) we obtain the limit of the steady state as follows,

$$\mathbf{d} > \frac{1}{\mathbf{P}_2} \left[ \sqrt{\left(\beta_1 - \frac{\mathbf{k}_1\mathbf{k}_2}{\mathbf{P}_1\alpha_1}\right) + \left\{\frac{1}{2}\left(\mathbf{k}_1 + \frac{\mathbf{k}_2}{\mathbf{P}_1\alpha_1}\right)\right\}^2} - \frac{1}{2}\left(\mathbf{k}_1 + \frac{\mathbf{k}_2}{\mathbf{P}_1\alpha_1}\right) \right] \quad (6)$$

Beyond this limit of  $\mathbf{d}$  the system became stable. Note that, if the system converges into  $x_{1,2} < -1$  and  $-1 < x_{1,2} < 1$  zone one can use the same method to get the limiting value of  $\mathbf{d}$ . The predicted limits of the steady state of the coupled system are for a typical case is shown in Fig 2 and it supports the numerical prediction.

#### IV. NUMERICAL SIMULATION RESULTS

The 4th order Runge-Kutta technique has been used for numerical integration of the state equations given in (4) to explore the dynamics of coupled Chua's circuit under non-identical dual channel coupling. Every time, we discard large amount of initial data to eliminate the transient behavior. The influences of coupling parameters ( $\mathbf{d}$ ,  $\mathbf{m}$ ,  $\mathbf{g}$ ) along with the conventional parameters like  $\alpha$ ,  $\beta$ ,  $\gamma$  on the dynamics of the coupled Chua's circuit has been examined through simulation.

To study the dynamics we initially take the two identical Chua circuits, for which we set  $\alpha_1 = \alpha_2 = \alpha = 10$ ,  $\beta_1 = \beta_2 = \beta = 18.43$ ,  $\gamma = 1$ ,  $\mathbf{d}_1 = \mathbf{d}_2 = \mathbf{d}$  and  $\mathbf{g}_1 = \mathbf{g}_2 = \mathbf{g} = 0$ , respectively. In this condition the two isolated Chua's circuits

show oscillatory behaviour. It is observed that the periodic oscillations of the coupled Chua's circuit under MFD coupling (i.e. single channel coupling ( $\mathbf{g} = 0$ )) would converge (both  $\mathbf{x}$  and  $\mathbf{y}$  state variables) to the trivial HSS (as predicted in section 3) with properly chosen  $\mathbf{d}$  and  $\mathbf{m}$  values i.e. AD occurs. This observation is shown in Fig.2. This is similar as reported in literature [34], [35].

Now with the application of second channel coupling (proper  $\mathbf{g}$  value) we observe that the trivial steady state of  $\mathbf{x}$  state variable becomes unstable and give birth of two new IHSS through symmetry breaking (i.e. OD is created) for low  $\mathbf{m}$  value. But, at the same time the  $\mathbf{y}$  state variable remains in the same HSS i.e. no OD is observed. Thus, we get a complex dynamics for a dual channel coupled system. Since, it is practically impossible to construct identical oscillators, because of the tolerance values of the circuit components. Therefore, we study the dynamics with non-identical parameters. For which we set the parameter as  $\alpha_1 = 10$ ,  $\alpha_2 = 0.95\alpha_1$ ,  $\beta_1 = 17.64$ ,  $\beta_2 = 1.05\beta_1$ ,  $\gamma = 1.05$ ,  $\mathbf{d}_1 = \mathbf{d}_2 = \mathbf{d}$  and  $\mathbf{g}_1 = \mathbf{g}_2 = \mathbf{g}$ . The observation is shown in Fig.3. Note that in Fig.3 in AD region  $\mathbf{x}$  is non zero which is the characteristic of a nontrivial AD, thus OD and possibly a nontrivial AD coexists as discovered in [14], [15]. It is also observed that the difference between two IHSS for  $\mathbf{x}$  state variable gradually increases as we increase the  $\mathbf{g}$  value but remain unaltered for variation of  $\mathbf{d}$  and  $\mathbf{m}$  values. We find that with the increasing value of direct diffusive coupling parameter ( $\mathbf{g}$ ) the AD domain tends to shrink while it enlarges the OD domain in the  $\mathbf{m} - \mathbf{d}$  parameter space. This fact is depicted in Fig.4. In Fig.4 beyond the quenched state we observe the system oscillates with multi-period. But since we are interested only in quenching phenomena we simply defined them as oscillatory state (OS).

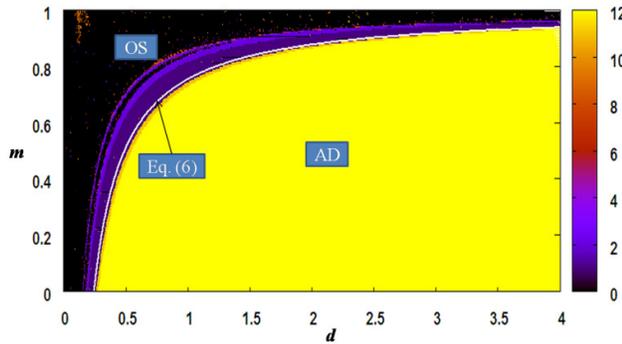


Fig. 2 Transition from the region of oscillatory state (OS) to amplitude death (AD) as observed through numerical simulation in parameter space ( $d - m$ ) for two identical Chua's circuits coupled under MFD coupling. The other system parameters are:  $\alpha_1 = \alpha_2 = \alpha = 10$ ,  $\beta_1 = \beta_2 = \beta = 18.43$ ,  $\gamma = 1$ ,  $d_1 = d_2 = d$  and  $g_1 = g_2 = g = 0$ , respectively. Here, OS defines the oscillatory dynamics with different periods (colour bar 0-9) and AD (colour bar 10) the quenched state. The limit of stable state as predicted analytically by equation (6) is shown by continuous white line

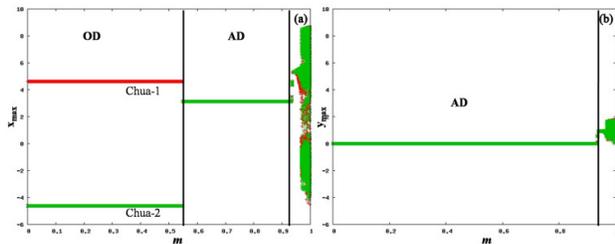


Fig. 3 (a) Transition of  $x$  state variable from the region of oscillation death state (OD) to amplitude death (AD) as observed through numerical simulation with  $m$  as control parameter under dual channel coupling. (b) Variation of  $y$  state variable with  $m$  as control parameter under dual channel coupling. No state transition is observed here. The other system parameters are:  $\alpha_1 = 10$ ,  $\alpha_2 = 0.95\alpha_1$ ,  $\beta_1 = 17.64$ ,  $\beta_2 = 1.05\beta_1$ ,  $\gamma = 1.05$ ,  $d_1 = d_2 = d$  and  $g_1 = g_2 = g$  respectively. The initial values of the coupled system are  $x_1(0) = 0.5$ ,  $y_1(0) = 0.04$ ,  $z_1(0) = 0.01$ ,  $x_2(0) = 0.05$ ,  $y_2(0) = 0.01$ ,  $z_2(0) = 0.05$

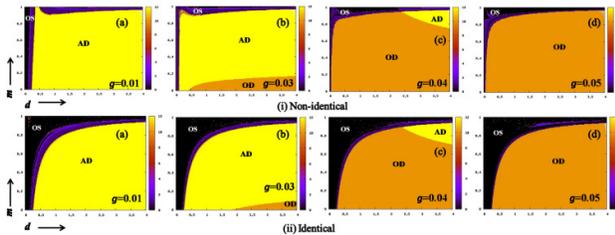


Fig. 4 The variation of AD, OD, and OS areas as observed through numerical simulation in  $m - d$  parameter space with  $g$  as control parameter. The other system parameters for (i) are:  $\alpha_1 = 10$ ,  $\alpha_2 = 0.95\alpha_1$ ,  $\beta_1 = 17.64$ ,  $\beta_2 = 1.05\beta_1$ ,  $\gamma = 1.05$ ,  $d_1 = d_2 = d$  and  $g_1 = g_2 = g$  respectively (i.e. both the systems are non-identical) and for (ii)  $\alpha_1 = \alpha_2 = 10$ ,  $\beta_1 = \beta_2 = 17.64$ ,  $\gamma = 1$ ,  $d_1 = d_2 = d$  and  $g_1 = g_2 = g$  respectively (i.e. both the systems are identical). In both cases the initial values of the coupled system are  $x_1(0) = 0.5$ ,  $y_1(0) = 0.04$ ,  $z_1(0) = 0.01$ ,  $x_2(0) = 0.05$ ,  $y_2(0) = 0.01$ ,  $z_2(0) = 0.05$

## V. EXPERIMENTAL STUDIES

Hardware circuits of the coupled Chua's circuit under non-identical dual channel coupling as shown in Fig. 1 are designed on a bread board using IC

TL082 (op-amp for Chua diode, inductor block and MFD coupling), capacitors and resistors etc. Here, we use a  $\pm 12$  volt power supply. The two Chua's circuits are designed using the following parameters [30]:  $R_{11} = R_{21} = 220$  ohm,  $R_{12} = R_{22} = 220$  ohm,  $R_{13} = R_{23} = 2.2$  kohm,  $R_{14} = R_{24} = 22$  kohm,  $R_{15} = R_{25} = 22$  kohm,  $R_{16} = R_{26} = 3.3$  kohm,  $R_{17} = R_{27} = 100$  ohm,  $R_{18} = R_{28} = R_{19} = R_{29} = 1$  kohm,  $R_{110} = R_{210} = 2.2$  kohm,  $C_{12} = C_{13} = C_{22} = C_{23} = 0.1\mu F$ ,  $C_{11} = C_{21} = 0.01\mu F$  and 2 k POT for each  $r_{11}$  and  $r_{21}$ . Here, the inductors in Chua's circuits are replaced by general impedance converters [36]. The effective inductances of general impedance converters are 22 mH each. The MFD coupling block has been constructed with  $R_1 = R_2 = R_4 = R_5 = 10$  kohm and  $R_3(\propto m)$ ,  $R_6(\propto \frac{1}{d})$  with 10 k POT. The direct coupling has been done by using a high value resistor  $R_7$ .

To explore the dynamics of such modified circuit we have first set  $r_{11} = 1.84$  kohm and  $r_{21} = 1.72$  kohm. With these values both the system show chaotic dynamics (Fig.5(a)). Then we apply MFD coupling. When we set  $R_3 = 1.59$  kohm (i.e.  $m = 0.32$ ) and  $R_6 = 2.35$  kohm (i.e.  $d \approx 0.78$ ) the chaotic dynamics converges to oscillation of period-1. At  $R_3 = 1.2$  kohm (i.e.  $m = 0.24$ ) the oscillatory behavior of both the systems completely vanishes and they converge to the same steady state (single dc line appears on the oscilloscope). Thus, AD occurs. These two results are shown in Fig.5(b) and Fig.5(c), respectively. In all these cases the direct coupling connection is taken as open (i.e.  $R_7 = \infty$ ).

Next we experimentally verify the influence of  $R_7$  ( $\propto \frac{1}{g}$ ) on the circuit. The AD state (single dc line) splits into two new steady states (two dc lines appear on the oscilloscope) even when we connect  $R_7$  with a very high value (300 kohm). Thus OD appears. The observation is shown in Fig. 6. The separation between the two dc line varies with the variation of  $R_7$ .

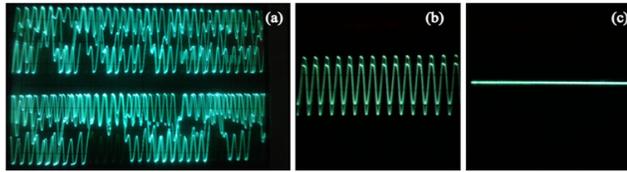


Fig. 5 Experimentally observed amplitude death in Chua's circuit. The time series traces of  $v_{11}$  and  $v_{21}$ ; (a) the chaotic dynamics in two isolated Chua's circuits for  $r_{11} = 1.84$  kohm and  $r_{21} = 1.72$  kohm; (b) Oscillation of period-1 dynamics of MFD coupled Chua circuit for  $r_{11} = 1.84$  kohm,  $r_{21} = 1.72$  kohm,  $R_3 = 1.59$  kohm and  $R_6 = 2.35$  kohm and (c) Occurrence of AD in the coupled system for  $r_{11} = 1.84$  kohm,  $r_{21} = 1.72$  kohm,  $R_3 = 1.2$  kohm and  $R_6 = 2.35$  kohm. The direct coupling connection is taken as open (i.e.  $R_7 = \infty$ )



Fig. 6 Experimentally observed transition from the amplitude death (AD) to Oscillation death (OD) for two Chua's circuits coupled through MFD coupling as well as direct coupling. The time series traces of  $v_{11}$  and  $v_{21}$ ; (a) the AD in coupled Chua's circuits for  $r_{11} = 1.84$  kohm,  $r_{21} = 1.72$  kohm,  $R_3 = 1.2$  kohm,  $R_6 = 2.35$  kohm and  $R_7 = \infty$ ; (b) occurrence of OD for  $R_7 = 300$  kohm; and (c) separation between two dc line increase with variation of  $R_7$

## VI. CONCLUSIONS

We have explored the quenching dynamics of the non-identical dual channel coupled Chua's circuits. Using detailed numerical simulations we have shown that an additional direct coupling through second channel can induce multi complex dynamics in the coupled system. It generates OD in a MFD coupled Chua's circuit, a transition between AD and OD and OD coexists with a nontrivial AD. Numerically we show that the fact is true for both identical and non-identical systems. It has been shown that OD appears in the system even when the direct coupling is very weak. We have also experimentally observed the generation of OD. This study includes a new dimension to the study of versatile nonlinear phenomena that one can find in most simple Chua's circuit. The introduction of additional coupling makes the system behavior more complex. So, the study can be extended to other chaotic systems to improve our understanding. We also hopeful apart from electronic circuits, the dynamics of such dual channel non-identically coupled system can be observed in engineering and biological systems and it may reveal the practical application of this transition.

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