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# A Study of Torsional Oscillations of a sphere in an Oldroyd-B Fluid

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**Abstract:** The Laminar flow of Oldroyd-B Fluid fluid due to torisional oscillations of a sphere has been studied. Expression for the velocity and the torque on the sphere have been found. The presence of elasticity in the fluid reduces the virtual mass of the sphere and the magnitude of the damping force acting on it.

***Keywords and phrases:*** *oldroyd B- fluid, torsional vibrations, torque, circular disc*

## 1. Introduction:

The problem of torsional oscillations of a sphere in a viscous fluid has been discussed by K R Raj gopal[4]. As many of the elastic properties of dilute polymer solutions can be detected and measured conveniently by observing suitable types of oscillatory flows, we consider here the theoretical study of a type of oscillating system which might prove useful when large quantities of fluid are available. In this system a sphere is suspended in an infinite expanse of Oldroyd-B Fluid B-model[7] elastic viscous fluid and made to perform torsional oscillations about one of its diameters.

## 2. Formulation of the Problem

We consider the primary flow due to the torsional oscillations of a sphere of radius '*a*' about a diameter. The linearized equation governing the motion under the assumption that the magnitude that of oscillation is very small[6] is given by

$$\rho\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(\frac{\partial w}{\partial t}\right)=\eta_0\left(1+\lambda_2\frac{\partial}{\partial t}\right)\left(\nabla^2-\frac{1}{r^2 sin^2\theta}\right)w \qquad (1)$$

Where w is the azimuthal component of velocity and

$$\nabla^2=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2 sin^2\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial}{\partial\theta}\right)$$

The motion is axisymmetric and all flow quantities are independent of $\varphi$ [8]

If the angular velocity of the sphere be Ω be ${}^{2\pi}/_{\sigma}$ be the period of oscillation, the boundary conditions are

$$w = \Omega\, a\, Sin\theta e^{i\sigma t} \; on\, r = a \tag{2}$$

Here

$$w \to 0 \; as\, r \to \infty$$

Therefore

$$w = \overline{w}e^{i\sigma t} \; where \;\; \overline{w} = \overline{w}(r,\theta)$$

Now (1) become

$$\left(\nabla^2 - \frac{1}{r^2 sin^2\theta}\right)\overline{w} - k^2\overline{w} = 0 \tag{3}$$

Where $k^2 = \frac{i\sigma\rho(1+\lambda_2 i\sigma)}{\eta_0(1+\lambda_1^{i\sigma})}$ (4)

The appropriate solution of (3) satisfying the boundary conditions is

$$w = e^{i\sigma t}\frac{\Omega a^3 Sin\theta}{r^2}e^{-k(r-a)}\left(\frac{1+kr}{1+ka}\right), k > 0 \tag{5}$$

Neglecting the terms of $O(|k|^3)$ on the assumption that the oscillations are small[3], the real part of (5) may be put as

$$w = \frac{\Omega a^3 Sin\theta}{r^2}(A\, Cos\sigma t + BSin\sigma t) \tag{6}$$

Where

$$A = 1 - \frac{1}{2}(r^2 - a^2)|k|^2 Cos\xi$$

$$B = \frac{1}{2}(r^2 - a^2)|k|^2 Sin\xi$$

$$|k|^2 = \frac{\sigma\rho}{\eta_0}\left(\frac{1+\lambda_1^2\sigma^2}{1+\lambda_2^2\sigma^2}\right)^{\frac{1}{2}} \tag{7}$$

$$\xi = \frac{\pi}{2} + tan^{-1}\lambda_1\sigma - \; tan^{-1}\lambda_2\sigma$$

The partial stress components may be calculated as $p_{\theta\phi} = 0$ (8)

$$p_{n\phi} = -\mu e^{i\sigma t}\frac{\Omega a^3 Sin\theta}{r^3}\left(\frac{3+3kr+k^2r^2}{1+ka}\right)e^{-k(r-a)} \tag{9}$$

Where $\mu = \frac{\eta_0(1+\lambda_2^{i\sigma})}{(1+\lambda_1 i\sigma)}$ (10)

## 3. Torque:

The torque on the sphere is given by

$$T = 2\pi a^3 \int_0^{\pi} p_{n\phi\delta=a} Sin^2\theta \, d\theta \tag{11}$$

Evaluation of this integral gives $T = -\frac{8}{3}\pi\mu a^3 \Omega e^{i\sigma t}\left(\frac{3+3ka+k^2a^2}{1+ka}\right)$ (12)

Neglecting terms of $O(|k|^2)$ and writing the real part alone equation (12) can be written as

$$T = M'\sigma\Omega(C\,Cos\sigma t - DSin\sigma t) \tag{13}$$

Where $M' = \frac{4\pi a^3 \rho}{3}$ is the mass of the fluid displaced by the sphere; and

$$C = \frac{6}{|k|^2} Cos\left(\frac{\pi}{2} + \xi\right) \tag{14}$$

$$D = 2a^2 + \frac{6}{|k|^2} Sin\left(\frac{\pi}{2} + \xi\right) \tag{15}$$

## 4. Numerical Results

The expressions for the velocity field and torque given respectively by the equations (6) and (13) may be thrown into non-dimensional forms

$$w = \Omega a\, Sin\theta(P\,Cos\sigma t + Q\,Sin\sigma t) \tag{16}$$

$$T = M'\Omega\sigma a^2(A^*\,Sin\sigma t + B^*\,Cos\,\sigma t\,) \tag{17}$$

Where $P = 1 - \frac{(R^2-1)}{2}|k|^2a^2\,Cos\frac{\xi}{R^2}$ where $R = \frac{r}{a}$ (18)

$$Q = \frac{\left(r'^2 - 1\right)}{2}|k|^2a^2\,Sin\frac{\xi}{R^2} \tag{19}$$

$$A^* = 2 + \frac{6}{a^2|k|^2}\,Cos\xi \tag{20}$$

$$B^* = \frac{6}{a^2|k|^2}\,Sin\xi \tag{21}$$

Values of the functions P & Q are entered in Tables (1) and (2). It may be observed that the values of P & Q increase steadily with increasing frequency due to the presence of elasticity in the fluid.

With a view to compare the torque T with that of the classical viscous fluid $T_0$ it has been thrown in the form

$$T/T_0 = A\,Cos\sigma t + (B - \alpha)\,Sin\sigma t \tag{22}$$

Where

$$T_0 = -\frac{8\pi a^3}{3}\,\Omega\eta_0 \tag{23}$$

$$\alpha = \frac{a^2\sigma\rho}{\eta_0} \tag{24}$$

$$A = 3\left(\frac{1+\lambda_2^2\sigma^2}{1+\lambda_1^2\sigma^2}\right)^{\frac{1}{2}} Cos(tan^{-1}\lambda_1\sigma - tan^{-1}\lambda_2\sigma) \tag{25}$$

$$B = 3\left(\frac{1+\lambda_2^2\sigma^2}{1+\lambda_1^2\sigma^2}\right)^{\frac{1}{2}} Sin(tan^{-1}\lambda_1\sigma - tan^{-1}\lambda_2\sigma) \tag{26}$$

The values of A and B are show in Table (3). It is interesting to note that the values of A steadily decrease with increasing frequency in all the cases of the elastico-viscous fluids considered, whereas the values of B increase up to a value of $\sigma$ and then decrease. It is evident from Table(3) that B has maximum values $B_{max} = 1.2, 1.326, 1.4917$ respectively in cases (ii) and (iii) and (iv) for $\sigma = 20, 10\ and\ 10$ approximately. Thus, greater the ratio ${}^{\lambda_2}/_{\lambda_1}$ higher is the frequency $\sigma$ at which $B_{max}$ occurs. From eq(15) it is noticed that the dimensionless velocity ${}^{w}/_{\Omega}\ a\ Sin\theta$ is composed of simple harmonic waves whose amplitudes increase with the distance from the sphere. Further the effect of an increase in the relaxation parameter $\lambda_1$ on the amplitude of each wave is to increase its magnitude and that of the retardation time parameter $\lambda_2$ is to decrease the same.

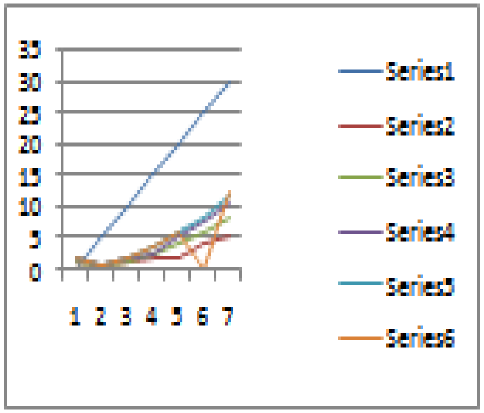
Also, from eq(22) it is found that the nondimensional torque ${}^{T}/_{T_0}$ is composed of two simple harmonic waves with amplitudes $A\ and\ (B-\alpha)$ travelling in the same direction or in opposite directions according as $B \lesseqgtr \alpha$. The effect of increasing the value of $\lambda_1$ is to reduce the magnitude of A and increase that of B while the effect of increasing the value of $\lambda_2$ is to increase the magnitude of A and decrease that of B.

To facilitate further interpretation, the expression for the torque T has been put in the form given by eq(16). The meaning of the two terms in the expression on the RHS of eq(17) for the torque can be seen as follows. The tangential force required to rotate the sphere of mass M in the absence of fluid stresses is $M\Omega\sigma a Sin\sigma t$. Eq(17) shows that, in addition, a further force $M'\Omega\sigma a\ Sin\sigma t$, in phase with the acceleration, is required. This arises because in the process of imparting torsional motion to the sphere, fluid is rotated as well. The quantity $M'$ may be called the virtual mass of the sphere and depends upon the frequency $\sigma$ in a very complicated way. The second term in the expression (17), $M'\Omega\sigma a\ Cos\sigma t$ always opposes the motion of the sphere and it thus a damping force out of phase with the acceleration. This force produces the decay of the torsional oscillations of the sphere if left free.

## 5. Graphical Representation

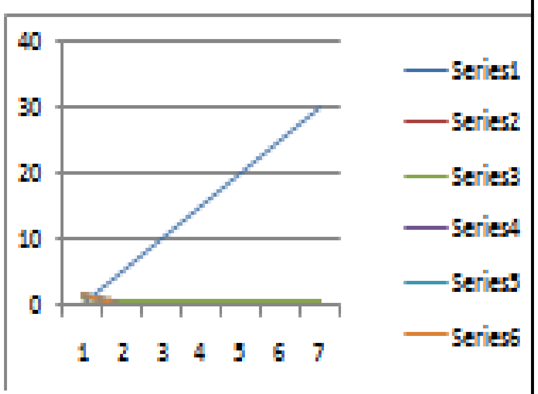
*Values of P in* $w = \Omega a \, Sin\theta (P \, Cos\sigma t + Q \, Sin\sigma t)$     *Fig* (1)

| σ/R | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 |
|---|---|---|---|---|---|
| 5 | 0.8468 | 0.7545 | 0.6946 | 0.6535 | 0.6241 |
| 10 | 1.2995 | 1.4801 | 1.5973 | 1.6777 | 1.7351 |
| 15 | 2.0392 | 2.6658 | 2.6818 | 3.3513 | 3.5507 |
| 20 | 2.3504 | 4.2779 | 5.0781 | 5.6268 | 6.0192 |
| 25 | 4.2892 | 6.2725 | 7.5597 | 8.4423 | 0.0250 |
| 30 | 5.7408 | 8.5986 | 10.4537 | 11.7256 | 12.6353 |



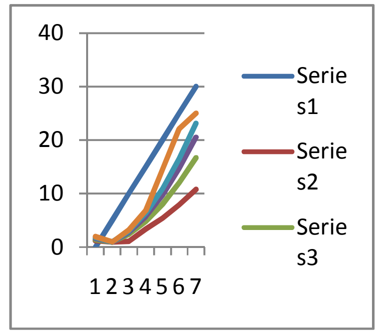

Case(i) $\lambda_1 = 0, \lambda_2 = 0$

| σ/R | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 |
|---|---|---|---|---|---|
| 5 | 0.6944 | 0.5102 | 0.3906 | 0.3806 | 0.0250 |
| 10 | 0.6944 | 0.5102 | 0.3906 | 0.3806 | 0.0250 |
| 15 | 0.6944 | 0.5102 | 0.3906 | 0.3806 | 0.0250 |
| 20 | 0.6944 | 0.5102 | 0.3906 | 0.3806 | 0.0250 |
| 25 | 0.6944 | 0.5102 | 0.3906 | 0.3806 | 0.0250 |
| 30 | 0.6944 | 0.5102 | 0.3906 | 0.3806 | 0.0250 |



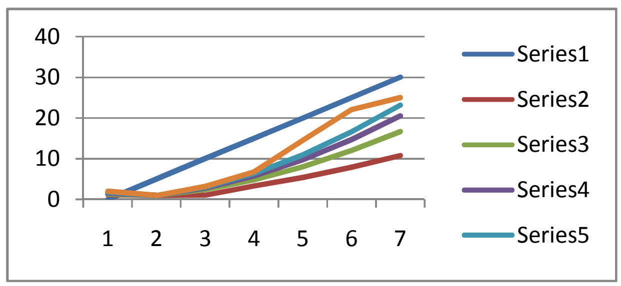
**Case(iii)** $\lambda_1 = 0.09, \lambda_2 = 0.01$

| σ/R | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 |
|---|---|---|---|---|---|
| 5 | 0.9992 | 0.9988 | 0.9985 | 0.9983 | 0.9981 |
| 10 | 1.0946 | 2.4500 | 2.8040 | 3.0467 | 3.2203 |
| 15 | 3.3839 | 4.8214 | 5.7543 | 6.3939 | 6.8515 |
| 20 | 5.3953 | 8.0455 | 9.7656 | 10.9449 | 14.5537 |
| 25 | 7.8840 | 12.0348 | 14.7289 | 16.5759 | 22.0951 |
| 30 | 10.7862 | 16.6870 | 20.5168 | 23.1425 | 25.0206 |




**Case(iii)** $\lambda_1 = 0.09, \lambda_2 = 0$

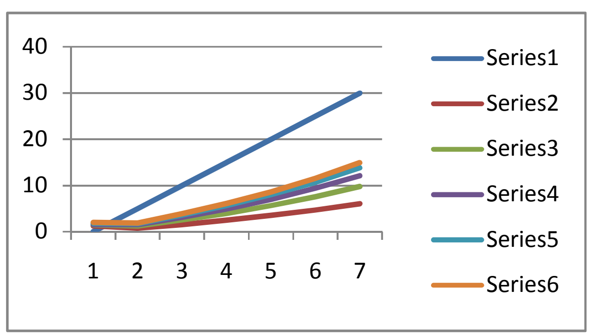
| σ/R | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 |
|---|---|---|---|---|---|
| 5 | 0.9992 | 0.9988 | 0.9985 | 0.9983 | 0.9981 |
| 10 | 1.0946 | 2.4500 | 2.8040 | 3.0467 | 3.2203 |
| 15 | 3.3839 | 4.8214 | 5.7543 | 6.3939 | 6.8515 |
| 20 | 5.3953 | 8.0455 | 9.7656 | 10.9449 | 14.5537 |
| 25 | 7.8840 | 12.0348 | 14.7289 | 16.5759 | 22.0951 |
| 30 | 10.7862 | 16.6870 | 20.5168 | 23.1425 | 25.0206 |




***Values of P in*** $w = \Omega a\, Sin\theta (P\, Cos\sigma t + Q\, Sin\sigma t)$ Fig(2)

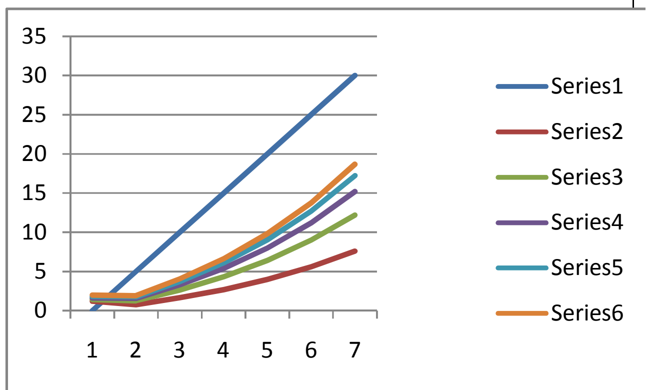
**Case(ii)** ) $\lambda_1 = 0.05, \lambda_2 = 0.01$

| σ/R | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 |
|---|---|---|---|---|---|
| 5 | 0.7715 | 1.2367 | 1.5386 | 1.7456 | 1.8937 |
| 10 | 1.5883 | 2.5460 | 3.1675 | 3.5937 | 3.8985 |
| 15 | 2.4934 | 3.9968 | 4.9726 | 5.6416 | 6.1201 |
| 20 | 3.5256 | 5.6515 | 7.0313 | 7.9772 | 8.6558 |
| 25 | 4.7181 | 7.5630 | 9.4095 | 10.6754 | 11.5809 |
| 30 | 6.0971 | 9.7734 | 12.1595 | 13.7954 | 14.9656 |



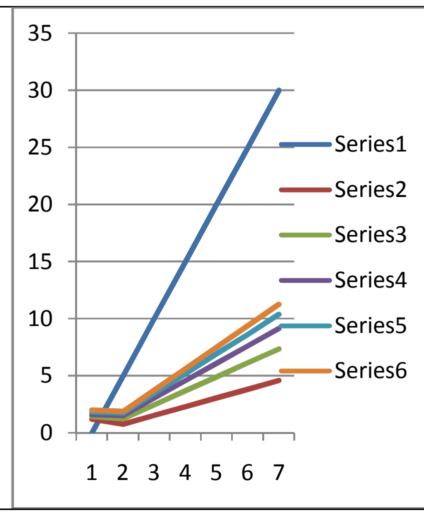
**Case(iii) )** $\lambda_1 = 0.09, \lambda_2 = 0.01$

| σ/R | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 |
|---|---|---|---|---|---|
| 5 | 0.7791 | 1.2489 | 1.5538 | 1.7629 | 1.9124 |
| 10 | 1.6488 | 2.6430 | 3.2882 | 3.7306 | 4.0470 |
| 15 | 2.6951 | 4.3201 | 5.3749 | 6.0980 | 6.6152 |
| 20 | 3.9951 | 6.4050 | 7.9688 | 9.0408 | 9.8077 |
| 25 | 5.6168 | 9.0036 | 11.2017 | 12.7088 | 13.7868 |
| 30 | 7.6109 | 12.2000 | 15.1785 | 17.2205 | 18.6812 |




**Case(i) & (iv)** $\lambda_1 = 0.09, \lambda_2 = 0$

| σ/R | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 |
|---|---|---|---|---|---|
| 5 | 0.7639 | 1.2245 | 1.5234 | 1.7284 | 1.8750 |
| 10 | 1.5278 | 2.4490 | 3.0469 | 3.4568 | 3.7500 |
| 15 | 2.2917 | 3.6735 | 4.5703 | 5.1852 | 5.6250 |
| 20 | 3.0556 | 4.8980 | 6.0938 | 6.9136 | 7.5000 |
| 25 | 3.8194 | 6.1224 | 7.6172 | 8.6120 | 9.3750 |
| 30 | 4.5833 | 7.3469 | 9.1406 | 10.3704 | 11.2500 |




## 5. Conclusions

It can be seen that $A^*$ has a maximum value of two, which is attained as the frequency $\sigma$ tends to infinity. As $\sigma$ tends to zero $A^*$ has minimum values of 2.0, 1.76, 1.52 and 1.46 respectively in cases (i) to (iv). The minimum value of $C_1$ decreases with the ratio $^{\lambda_2}/_{\lambda_1}$. Further, it may be noticed that the value of $A^*$for the Newtonian fluid, i.e., Case(i) is very much larger than any one of the elastic-viscous fluid (ii) and (iv). The function $A^*$ which is constant for the Newtonian fluid takes a curved shape for an elastic-viscous fluid. Further, the curve rises steeply up to a value of $\sigma = 20$

Fig.2. shows the variation of $A^*$ against $\sigma$ in all the above mentioned four cases. It can be seen that in each case $A^*$ tends to infinity as $\sigma$ tends to zero. It tends to zero for large $\sigma$. It is also noticed that for

each value $\sigma$, $B^*$ is smaller the lesser the ratio ${}^{\lambda_2}/_{\lambda_1}$. For example, when $\sigma = 25$ the value of $B^*$ for Newtonian fluid is nearly 100% larger than the value of $C_2$ for the elastic-viscous fluid of case(ii)

It may be observed that the presence of elasticity in the fluid has the effect of reducing the virtual mass of the sphere and the magnitude of the damping force acting on the sphere

## 6. References

[1]. Ghosh, A.K. Datta, S.K. and Sen, P(2014) A Note on hydro magnetic flow of an Oldroyd B-fluid near infinite plate induced by Half rectified sine pulses, *Open Journal of Fluid Dynamics*, 4, 226-240

[2]. Sneddon Ian: The use of integral Transforms, Mc.grawhill edition

[3]. Ghosh, A.K. (1965) Flow of Viscous Liquid Set up between Two Co-Axial Cylinders Due to Pulses of Longitudinal Impulses Applied on the Inner Cylinder. *Journal of Science and Engineering Research*, **9**, 222.

[4]. K. R. Rajagopal,(1982) A note on unsteady unidirectional flows of a non-Newtonian fluid, Int. *Journal of Non-Linear Mech.* **17**, 373.

[5]. T.Hayat (2004) The flow of a viscoelastic fluid on an oscillating plate, *Journal of Applied Mathematics and Mechanics,* 84, 65-70

[6]. Vatsala Mathu (2014) Exact solution for the flow of Oldroyd-B fluid due to constant shear and time dependent velocity, *IOSR Journal of Mathematics,* Volume 10, Issue 4 Ver. II (Jul-Aug. 2014), PP 38-45

[7]. C. Fetecau and K Khannan (2005) A Note on unsteady flow of an Oldroyd B-fluid, *International Journal of Mathematics & Mathematical Sciences* 2005:19 3185–3194

[8]. N. D. Waters and M. J. King, (1971) The unsteady flow of an elastico-viscous liquid in a straight pipe of circular cross section*, J. Phys. D: Appl. Phys*. **4**, no. 2, 204–211.

[9]. Manuel A. Alves, Paulo J. Oliveira, Fernando T. Pinho (2003), Benchmark solutions for the flow of Oldroyd-B and PTT fluids in planar contractions, *Journal of Non-Newtonian Fluid Mechanics*, 110 (2003) 45-75

[10]. D. Vieru, M. Nazar, Corina Fetecau, C. Fetecau, New exact solutions corresponding to the first problem of Stokes for Oldroyd-B fluids, *International Journal of Computers and Mathematics with Applications* 55 (2008) 1644–1652