

Tuning of a PID Controller to Control Overdamped Second-order-like Processes having Small Natural Frequencies

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Abstract:

This paper presents an effective tuning approach for PID controllers used to control overdamped second-order-like processes having small natural frequency up to 1 rad/s and damping ratio from 1 to 5. The MATLAB program is used through its optimization toolbox to tune the controller using the ITAE error-based criterion without any functional constraints. The tuning technique used is compared with another technique and the effectiveness of the controller is investigated. The effectiveness of the used optimization technique is outlined considering the possibility of kick elimination associated with PID controllers and providing a robust controller.

Keywords — PID controller, controller tuning, overdamped second order process having small natural frequency, control system performance.

I. INTRODUCTION

The PID controller family belongs to the first generation of PID controllers. They have some problems, mainly the kick phenomena where researchers are paying more efforts of overcome it through controller tuning. This paper is an effort aiming at improving the control system performance using PID controllers and second order overdamped processes having small natural frequency.

Girirajkumar, Kumar and Anantharaman (2010) studied the tuning of a PID controller for a first-order-delayed industrial process using particle swarm optimization. They used the ITAE error criterion, tuned the controller and achieved a step time response with 13 % maximum overshoot and 1980 s settling time [1]. Tandon and Kaur (2011) investigated the use of a PID controller tuned using genetic algorithm to control the concentration of the stream leaving a three-tank system. They used the error criteria MSE, IAE and ITAE as objective function for optimization technique used. They attained a maximum overshoot of 40.4 % and a settling time of 3 s when used an ITAE error criterion [2].

Chiha, Ghabi and Liouance (2012) studied the tuning of a PID controller using the multi-objective differential evolution technique. They compared

with Ziegler-Nichols method and outlined that the differential evolution technique provided better control system performance [3]. Hassaan (2014) investigated the tuning of PID controllers when used with second order overdamped processes. He used the MATLAB optimization toolbox and an ISE error criterion to tune the PID controller. The second order processes considered had damping ratio from 1 to 10 and natural frequency from 2.5 to 15 rad/s. He compared his tuning technique with the tuning technique based on using the standard forms [4]. Ibrahim, Amuda, Mohammed and Kareem (2013) investigated the performance of using a PID controller to control a delayed first-order process using Hagglund-Astrom, Cohen-Coon and Ziegler-Nichols tuning techniques. They showed that the Ziegler-Nichols algorithm provided the best control system performance with minimum rise and settling times [5].

Singh and Joshi (2017) reviewed the use of a number of tuning techniques for a PID controller used to control a delayed first-order process. Their review handled the Ziegler-Nichols, Cohen-Coon, calculus, linear programming, dynamic programming, stochastic programming, stimulated annealing, evolutionary algorithms, genetic algorithms, particle swarm optimization, ant colony optimization, artificial bee colony and teaching learning based optimization tuning techniques [6].

Hassaan (2019) presented the tuning of a PID controller used to control a second-order underdamped process having a damping ratio up to 0.9 a natural frequency up to 2.25 rad/s. He could eliminate the kick associated with the PID controller and relatively fast step time response for reference input tracking. He compared with the minimum ITAE standard forms as a tuning technique [7].

Turan (2021) investigated the improved optimum PID controller tuning used with a second-order and third-order processes (as examples) by minimizing the settling time and maximum overshoot. Through simulation his approach could reduce the settling time to 0.949 s for the second-order process and 2.161 s for the third-order process and a maximum percentage overshoot of 15 % for both processes [8]. Jiang and He (2022) performed research on a nonlinear DC electronic load system based on PSO-PID algorithm. They adopted a parameter self-tuning PID controller based on particle swarm optimization. The process (DC electronic load system) was modelled as a first-order one. They compared the application of their proposed controller with a traditional PID controller and illustrated the improvement obtained using the PSO-PID controller [9]. Nguyet and Ba (2023) studied the application of a neural flexible PID controller for a task-space control of robotic manipulators. They proposed an intelligent control of a 2DOF robotic manipulator based on a conventional PID structure [10].

II. PROCESS

The controlled process is second-order-like process having the transfer function, $G_p(s)$:

$$G_p(s) = \omega_n^2 / (s^2 + 2\zeta\omega_n s + \omega_n^2) \quad (1)$$

Where:

- ω_n = process natural frequency
- ζ = process damping ratio (> 1)

III. THE PID CONTROLLER

The structure of a PID controller for the control of a linear process is set in the forward path with the process as shown in Fig.1 [11]. This structure is for a set-point tracking control system.

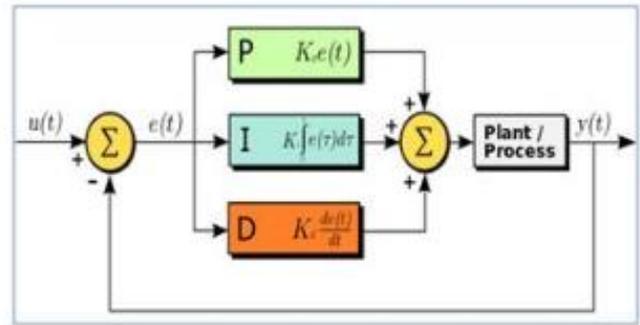


Fig.1 Structure and location of a PID controller controlling a process [11].

The PID controller has a transfer function $G_c(s)$ given by:

$$G_c(s) = K_{pc} + (K_i/s) + K_d s \quad (2)$$

Where:

- K_{pc} is the proportional gain
- K_i is the integral gain
- K_d is the derivative gain

IV. CONTROL SYSTEM TRANSFER FUNCTION

The closed loop transfer function of the control system is obtained using the block diagram of Fig.1 with unit feedback elements and Eqs.1 and 2 and given by:

$$M(s) = (b_0 s^2 + b_1 s + b_2) / (a_0 s^3 + a_1 s^2 + a_2 s + a_3) \quad (3)$$

Where:

$$\begin{aligned} b_0 &= \omega_n^2 K_d \\ b_1 &= \omega_n^2 K_{pc} \\ b_2 &= \omega_n^2 K_i \\ a_0 &= 1 \\ a_1 &= 2\zeta\omega_n + \omega_n^2 K_d \\ a_2 &= \omega_n^2 (1 + K_{pc}) \\ a_3 &= \omega_n^2 K_i \end{aligned}$$

V. CONTROLLER TUNING AND SYSTEM TIME RESPONSE

The controller parameters are tuned as follows:

- The control and optimization toolboxes of MATLAB are used to assign the three parameters of the controller [12].
- The integral of the time multiplied by the absolute error of the control system (ITAE) is chosen as an objective function for the optimization process.
- The optimization command '*fminunc*' is used to minimise the objective function without using any functional constraints [12].
- The step response of the closed-loop control system is plotted using the command '*step*' of MATLAB [13].
- The controller is tuned using the above approach for an overdamped second process with assigned damping ratio and natural frequency.
- The time-based specifications of the closed-loop control system mainly the maximum percentage overshoot is calculated using the maximum step time response and the steady-state time response of the closed loop control system. The settling time is evaluated from the step time response of the system and the $\pm 2\%$ band around the steady state time response.
- Process parameters covered in the present analysis:
 - Damping ratio: $1 \leq \zeta \leq 5$
 - Natural frequency: $0.1 \leq \omega_n \leq 1.0$ rad/s
- The tuned parameters of the controller are given in Table 1 through Table 4 depending on the process parameters values.

TABLE 1
TUNED CONTROLLER PARAMETERS FOR A DAMPING RATIO OF 1

ω_n rad/s)	K_{pc}	K_i	K_d
0.1	1.1163	0.0705	0.2009
0.2	1.1145	0.1386	0.2013
0.3	1.1146	0.2049	0.1981
0.4	1.3973	0.3109	0.2297
0.5	1.3793	0.3941	0.2169
0.6	3.1379	0.9764	1.8922
0.7	2.2574	0.8276	0.8422
0.8	2.2995	1.0270	1.4933

0.9	2.5912	1.2975	1.2722
1	2.9587	1.6339	1.4650

TABLE 2
TUNED CONTROLLER PARAMETERS FOR A DAMPING RATIO OF 2

ω_n rad/s)	K_{pc}	K_i	K_d
0.1	5.0097	0.1363	0.1991
0.2	4.5488	0.2605	0.9324
0.3	4.1670	0.3622	0.9013
0.4	5.5315	0.6188	0.9516
0.5	4.4179	0.6259	1.0251
0.6	4.3962	0.7596	1.0009
0.7	3.3937	0.7154	1.0053
0.8	4.4937	1.0936	1.5366
0.9	3.3819	0.6600	1.0252
1	40.8023	8.2503	9.6024

TABLE 3
TUNED CONTROLLER PARAMETERS FOR A DAMPING RATIO OF 3

ω_n rad/s)	K_{pc}	K_i	K_d
0.1	9.0035	0.1824	0.2495
0.2	9.0124	0.3635	0.1960
0.3	9.2152	0.5609	0.1896
0.4	8.4880	0.6783	0.4618
0.5	8.4863	0.8466	0.4811
0.6	8.3550	0.9982	0.4104
0.7	8.2322	1.1541	0.5737
0.8	8.4418	1.3457	0.3422
0.9	8.4440	1.5149	0.2817
1	8.4339	1.6811	0.2876

TABLE 4
TUNED CONTROLLER PARAMETERS FOR A DAMPING RATIO OF 4

ω_n rad/s)	K_{pc}	K_i	K_d
0.1	9.5732	0.1355	0.1920
0.2	9.3416	0.3056	0.1920
0.3	9.1846	0.4299	0.2144
0.4	8.4330	0.5136	0.4425
0.5	8.3309	0.6739	0.4515
0.6	8.1089	0.7862	0.5495
0.7	8.2064	0.8013	0.6380
0.8	8.1302	0.9652	0.6267
0.9	6.0128	0.7865	0.6018
1	6.0035	0.8719	0.6022

TABLE 5
TUNED CONTROLLER PARAMETERS FOR A DAMPING RATIO OF 5

ω_n rad/s)	K_{pc}	K_i	K_d
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0.1	8.0002	0.0951	0.2000
0.2	9.6071	0.2302	0.1993
0.3	8.0034	0.2854	0.2000
0.4	6.5002	0.3046	0.2000
0.5	5.0014	0.2899	0.2000
0.6	3.5058	0.2582	0.1995
0.7	3.5007	0.2952	0.2000
0.8	3.2002	0.2990	0.2000
0.9	2.5136	0.2473	0.1995
1	2.1632	0.2274	0.1922

Comments on the Tuning Tables:

- The tuning approach used in this work produces a control system performance with maximum percentage overshoot less than 1.74 %.
- The settling time is less than 40 seconds.
- The tuned controller parameters corresponding to a process with 1 rad/s natural frequency and unit damping ratio produces a step time response of unit second settling time and zero overshoot.
- As an application of the present tuning technique of the PID controller when used with overdamped second-order-like processes we consider a second order process with 1 rad/s natural frequency and a 1.0 damping ratio (critical damping).
- Using Table 1, the tuned controller parameters in this case are: $K_{pc} = 2.9587$, $K_i = 1.6339$ and $K_d = 1.4650$.
- The unit step response of the control system incorporating the tuned PID controller and the overdamped second-order-like process is shown in Fig.2.

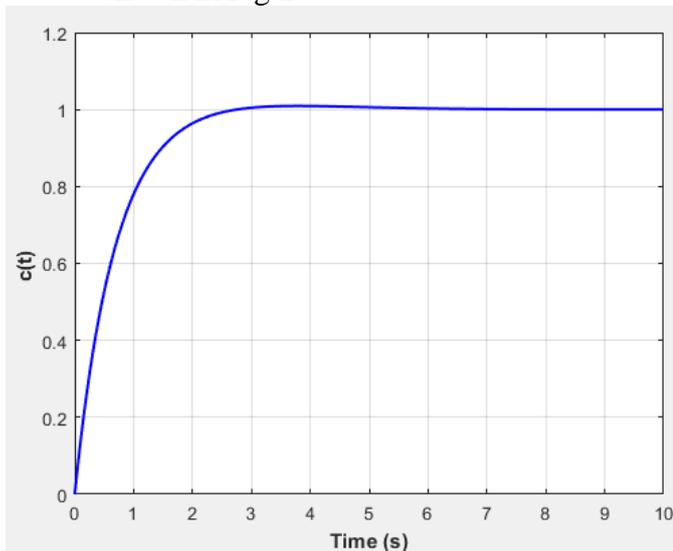


Fig.2 Unit step time response of the PID controlled overdamped second order process.

- Using the tuning technique applied in this paper, the characteristics of the closed control system incorporating the PID controller are as follows:
 - Maximum percentage overshoot: 0.91 %
 - Settling time: 2 s
 - Gain margin: ∞
 - Phase margin: 87.8°

VI. COMPARISON WITH MINIMUM ITAE STANDARD FORMS TUNING

- The parameters of the PID controller is tuned using the minimum ITAE standard forms of Graham and Lathrop [14]. The resulting controller parameters are:
 - $K_{pc} = 5.8532$
 - $K_i = 1.6340$
 - $K_d = 1.4982$
- The unit step time response of the closed loop control system incorporating the PID controller and the process with $\zeta = 1$ and $\omega_n = 1$ rad/s is shown in Fig.3. using both tuning techniques.

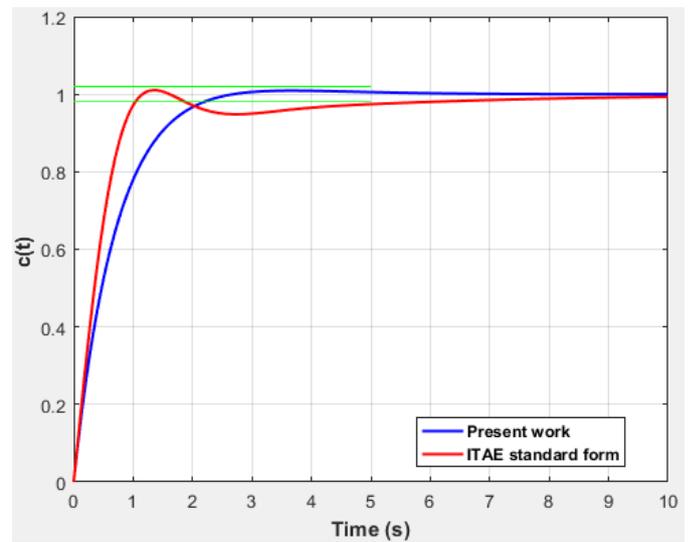


Fig.3 Unit step time response using two tuning techniques.

- The characteristics of the control system using the two tuning techniques are compared in Tables 6.

TABLE 6
CHARACTERISTICS COMPARISON
USING TWO TUNING TECHNIQUES

Tuning technique	Present tuning	ITAE standard forms
OS _{max} (%)	0.918	1.046
T _s (s)	2.25	5.00
GM (dB)	∞	∞
PM (degrees)	87.8	72.0

VII. CONCLUSIONS

- The PID controller was tuned for use with second-order-like overdamped processes with damping ratio between 1 and 5 and natural frequency from 0.1 to 1 rad/s.
- It was tuned using the MATLAB optimization toolbox.
- The ITAE error criterion was used as an objective function for the optimization problem of the controller tuning.
- The tuning technique used was compared with another one using the minimum ITAE standard forms.
- The time response kick was almost eliminated using the proposed tuning technique for the considered process parameters.
- The tuned PID controller has improved the performance of the closed loop control system compared with the used tuning technique.
- With infinite Gain Margin and 87.8 degrees Phase Margin, the tuning technique presented in this paper produced robust controller when used with the overdamped second order-like process having small natural frequency.

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BIOGRAPHY



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- Emeritus Professor of System Dynamics and Automatic Control.
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