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RESEARCH ARTICLE OPEN ACCESS

Application of Eulerian & Hamiltonian Cycle in Travelling

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Abstract:

Graph theory is a branch of mathematics which has wide application in travelling & Operation research . In this article we have discussed the application of Hamiltonian cycle & eulerian cycle in travelling.

Keywords— **Eulerian cycle, Hamiltonian cycle, cycle, graph.**

I. Introduction

In the first part of article , we discuss the some definition related to Hamiltonian graph. Then we solve travelling salesman Problem using Hamiltonian cycle. In the second part of article , we discuss the some definition related to Eulerian graph & solve Chinese postman problem using Eulerian cycle

II. Preliminaries With An Example

A. Graph

Graph is a diagram which is made by two sets ,vertex set & Edges set. Graph is denoted by G ,given by

G=G(V,E),where V= set of vertex & E= set of edges.

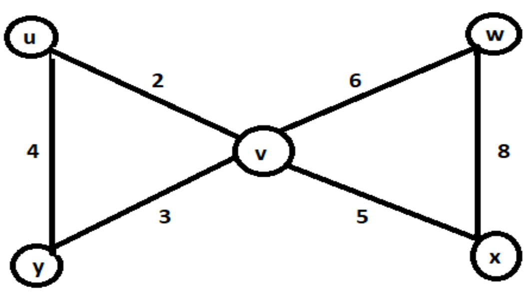

Fig.1 Example of graph

Here V={u,v,w,x,y} & E={2,4,6,3,5,8}

B. Hamiltonian Graph

1) ***Eulerian Cycle:*** Eulerian cycle is a cycle which contains every edge of given graph (Edges repetition is not allowed but, vertex

First we discuss the Hamiltonian cycle then definition of Hamiltonian graph.

1) ***Hamiltonian Cycle:*** Hamiltonian cycle is a cycle which passes through every vertex exactly once except initial vertex(Obviously ,repetition of edges is not allowed.)
2) ***Hamiltonian Graph:*** If given graph G contains Hamiltonian cycle then G is said to be Hamiltonian graph.

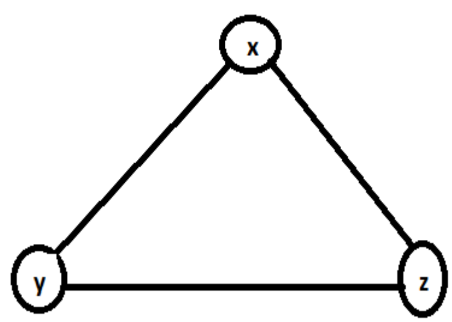

Fig. 2 Example of an Hamiltonian graph

C. Eulerian Graph

First we discuss the Eulerian cycle then definition of Eulerian graph.

repetition is allowed)

2) ***Eulerian Graph:*** If given graph G contains Eulerian cycle then

G is said to be Eulerian graph.

Ex.

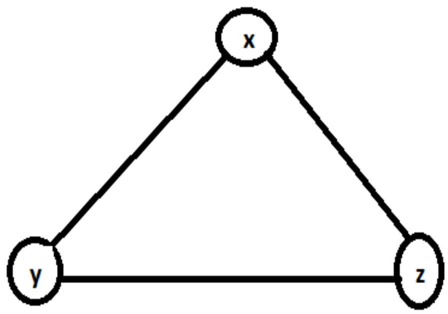

Fig. 3 Example of an Eulerian graph

D. *Weighted Graph*

In which every edge has assigned a positive real number , that number is called weight of that edge.

III. Travelling Salesman Problem

This problem is related to Hamiltonian cycle , Salesman is required to visit a no. of cities during the trip with covering least possible total distance. In which we use given cities as a vertex , different road between cities as a edges & distance between cities as a weight of that edge between vertices. For this equivalent concept in graph theory is require to find a Hamiltonian cycle of least possible total weight in a weighed graph.

ex:-

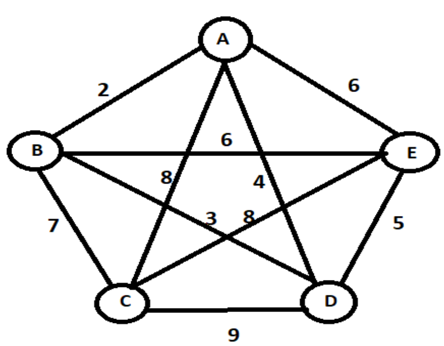

Fig.4 Graph represented 5 cities with their distance

Suppose A,B,C,D & E are 5 cities & There are many routes from one city to other city. Suppose to find shortest route from city A to A crossing all remaining cities.

A. *Trick*

given data in the graph represented as,

TABLE I

	A	B	C	D	E
A	0	2	8	4	6
B	2	0	7	3	6
C	8	7	0	9	8
D	4	3	9	0	5
E	6	6	8	5	0

Now to give the assignment in above table such that each row or column has only one assignment.

First we avoid column A because in this column give the assignment in the last. Now go to the 1^{st} row ‘A’ in which there are four elements 2,8,4 & 6 out of which 2 is a smallest number.Therefore we give the assignment to ‘2’.remember that assignment completed column is avoid for the next assignment. Therefore we avoid column ‘B’.minimum no.is in B therefore next we go to the row ‘B’. In which there are three elements 7,3 & 6 out of which 3 is a smallest number.Therefore we give the assignment to ‘3’.now also we avoid column ‘D’. minimum no.is in D therefore next we go to the row ‘D’. In which there are two elements 9 & 5 out of which 5 is a smallest number.Therefore we give the assignment to ‘5’. we avoid column ‘E’. minimum no.is in E therefore next we go to the row ‘E’. In which only one element 8 is remaining ,therefore we give the assignment to ‘8’ .8 is in column C therefore lastly we go to the row C and give the assignment to ‘8’ in the 1^{st} ly avoided column.

In this way we obtain the Hamiltonian cycle is A-B-D-E-C-A with minimum weight = 2+3+5+8+8=26

i.e. It is equivalent to find shortest path from city A to A crossing all cities once is A-B-D-E-C-A with shortest distance is 26 km.

TABLE III

	A	B	C	D	E
A	0	2	8	4	6
B	2	0	7	3	6
C	8	7	0	9	8
D	4	3	9	0	5
E	6	6	8	5	0

B. *Remark*

If more than one same smallest element at a time then we can choose any one ,find Hamiltonian cycle with their weight .After this we choose other one also find Hamiltonian cycle with weight & so on.

Finally we choose Hamiltonian cycle with least possible weight between them.

Ex.

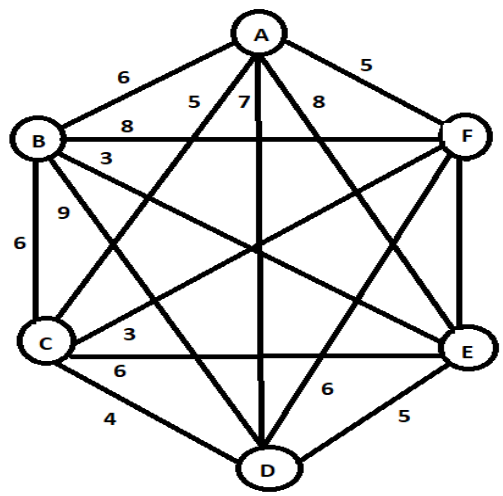

Fig.5 Graph represented 6 cities with their distances

given data in the graph represented as,

TABLE IIIII

	A	B	C	D	E	F
A	0	6	5	7	8	5
B	6	0	6	9	3	8
C	5	6	0	4	6	3
D	7	9	4	0	5	6
E	8	3	6	5	0	7
F	5	8	3	6	7	0

There are two minimum 5 & 5 in the 1st row A.

1) Case-I

TABLE IV

	A	B	C	D	E	F
A	0	6	5	7	8	5
B	6	0	6	9	3	8
C	5	6	0	4	6	3
D	7	9	4	0	5	6
E	8	3	6	5	0	7
F	5	8	3	6	7	0

Hamiltonian cycle is , A-C-F-D-E-B-A with weight=5+6+3+5+6+3=28

2) Case-II:

TABLE V

	A	B	C	D	E	F
A	0	6	5	7	8	5
B	6	0	6	9	3	8
C	5	6	0	4	6	3
D	7	9	4	0	5	6
E	8	3	6	5	0	7
F	5	8	3	6	7	0

Hamiltonian cycle is , A-F-C-D-E-B-A with weight =5+4+6+3+3+5 =26

Therefore, required Hamiltonian cycle is, A-F-C-D-E-B-A with

minimum weight =5+4+6+3+3+5 =26 .

IV. Fleury's Algorithm

It is used for finding Eulerian cycle from the given graph.

A. *Rules*

First we choose any vertex from the graph.

1) Start walking along the edges such that each edge cross exactly once.
2) Remove the edge if we traversed along that edge & also remove the isolated vertices in this process.
3) This procedure stops when we return to the starting vertex by traversing all edges & crossing all vertices.

To find Eulerian cycle from the following graph:

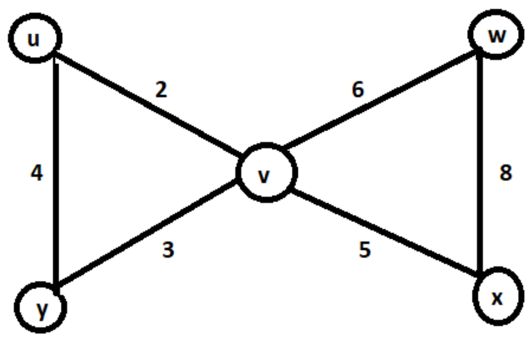

Fig.6 Example of a eulerian graph

Here u, v, w, x & y are vertices and 2,4,3,5,6 & 8 are edges.

There are many Eulerian cycles.

Eulerian cycles starting from vertex u :

1) u-2-v-5-x-8-w-6-v-3-y-4-u
2) u-4-y-3-v-6-w-8-x-5-v-2-u
3) u-2-v-6-w-8-x-5-v-3-y-4-u
4) u-4-y-3-v-5-x-8-w-6-v-2-u

similarly , Eulerian cycles starting from vertex v :

1) v-2-u-4-y-3-v-6-w-8-x-5-v
2) v-3-y-4-u-2-v-5-x-8-w-6-v
3) v-6-w-8-x-5-v-2-u-4-y-3-v
4) v-5-x-8-w-6-v-3-y-4-u-2-v
5) v-2-u-4-y-3-v-5-x-8-w-6-v
6) v-3-y-4-u-2-v-6-w-8-x-5-v
7) v-6-w-8-x-5-v-3-y-4-u-2-v
8) v-5-x-8-w-6-v-2-u-4-y-3-v

Similarly we can find Eulerian cycle from vertex w ,x & y.

V. Chinese Postman Problem:

In this problem , a postman wishes to deliver his letters with covering the least possible total distance & returning to his starting location. To solve Chinese Postman problem is equivalent to find shortest Eulerian cycle containing all edges of a connected graph, such an eulerian cycle can be found using Fleury's algorithm.

A. Remark

If graph is not eulerian then we introduce the process of duplication of an edge.

" An edge 'e' is said to be duplicated when it's ends are joined by a new edge with the same weight as 'e'".

Ex.

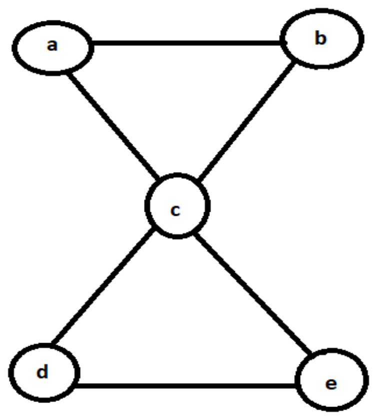

Fig.7Example of a eulerian graph

Here given graph is eulerian .now to find eulerian cycle using Fleury's algorithm as, a-b-c-d-e-c-a .

VI. Conclusion

In this article , we can find the minimum distance of the route by using Hamiltonian as well as eulerian cycle.

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