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# Collateral Portfolio Optimization Using Mixed Integer Linear Programming

Lurdu Matha reddy , Kuni Reddy
Princeton - NJ

## Abstract:

Before the Global financial crisis in 2008, financial institutions mitigating the credit default risk were mostly considered as siloed functions and sometimes completely ignored. These caused systemic collapse triggering governments across the globe to impose strict regulations. They forced them to risk and collateralize the exposures in regular cycles.

Counterparties that participated in OTC (Over the counter) derivative trades weredetermined by the ISDA CSA (Credit Support Annex) which captured Margining intervals, providedcollateral, and other fine-tuned rules for the risk exposures.

The financial organizations, which failed to follow the regulations,were heavily fined. Most of them lacked an organizational collateral view that shelled out the cash to satisfy the regulatory requirement.Besides, they represented other valuable collateral that faced a liquidity crisis.

The new optimization model described in this paper was successfully applied at the organizational level. Moreover, collateral optimization involved all the practical constraints like the number of substitutions, the opportunity cost of an asset, REV REPO cost of an asset, settlement cycles, and the bond borrows which yielded millions of dollars for the organizations.

**Keywords-** Collateral Optimization, CTD, Portfolio Optimization, LPP, MIP, Linear Programming, OTC, Derivatives, credit risk

## I. INTRODUCTION

Financial organizations that participate in OTC derivative trading are exposed to counterparty risk based on the movement of the trade. After the 2008 crisis, Federal regulations mandated organizations to collateralize them. ISDA agreement provides an Annex called CSA (Credit Support Annex) which states the standard format of specifyingthe needs of the counterparties.The acceptable ratings of the issuer and issue of the collateral, the amount of haircut applied on the valuation of the collateral, the allowed settlement cycles for receiving and depositing the collateral, and the time intervals to callout necessitated the risk for collateralization.

Since the trade market price movement happens in real time,the organizations need to calculate the risk exposure daily or based on agreed time interval according to CSA. If the organizationsare at risk, they need to issue a margin call to the counterparty and get the collateral covering the risk. The organizations follow the MTM (Mark to Market) process when calculating the risk exposure where theyconsider the list of trades and the value of the collateral exchanged with the counterparty. Based on the risk of the organization, either issues or receives a margin call to/from the counterparty. If the counterparty issues a margin call,itneeds collateral covering the value of the margin call.

When deciding the type of collateral to be issued, the organization needs to consider various factors like the eligible collateral, its type of haircut, its value, liquidity, availability, borrowing cost, and the opportunity cost. Part of the CSA agreement involves the counterparties agreeing on the re-hypothecation (Using the collateral received for some other purpose). While considering the necessary collateral, the organizations can recall it from some other counterparties by substitution.

Most of the organizations manage them by issuing cash or picking eligible collateral. Few of them createfew solutions by enlisting them from the book by price and picking the cheapest one according to calls from the receiving margins.

All these approaches do not have a holistic view of the organizational portfolio of the asset/collateral and miss the opportunity to utilize the collateral best. Itis increasing pressure on the funding desk to reserve budget for its borrowing while there is a lot of unused collateral within the organization and casing that emit the opportunities.

The new optimization model utilizes the collateral/asset portfolio of an organization combined with the machine learning clustering techniques that are explained in this paper. The model is successfully deployed in one of the big investment banks yielding millions of dollars savings. It is effective considering all the practical constraints that are deployed in Hadoop big data platform.

**Definitions**

*A. Derivative*

It is financial security with a value that is relied upon or derived from an underlying asset or group of assets.

*B. OTC*

Over-the-counter (OTC) is security traded in some context other than on a formal exchange such as the New York Stock Exchange (NYSE).

*C. Counterparty Risk*

Counterparty risk is the peril to each party of a contract that the counterparty will not meet its contractual obligations. Business Process Reengineering.

*D. ISDA*

An ISDA (International Swaps and Derivatives Association) Master Agreement is the standard document that is regularly used to govern over-the-counter derivatives transactions.

*E. CSA*

A credit support annex (CSA) provides credit protection by setting forth the rules governing the mutual posting of collateral.

*F. MTM*

Mark to market (MTM) is a measure of the fair value of accounts that can change over time, such as assets and liabilities.

*G. Margin Call*

A margin call occurs when a broker directs that an investor deposits additional money or securities so that the margin account is brought up to the minimum maintenance margin. A margin call occurs when the account value falls below the broker's required minimum value.

*H. CTD*

Cheapest to Deliver (CTD) is the process of selecting the cheapest collateral to issue a response to the margin call.

*I. LPP*

Linear programming Problem (LPP, also called linear optimization) is a method of achieving the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships.

*J. Clustering*

Clustering is an allocation of a set of probes into subsets (called clusters) so that observations in the same cluster are same in some sense. Clustering is a process of unsupervised learning and a common procedure for statistical data analysis used in many fields.

## II. DESIGNING A PORTFOLIO OPTIMIZATION PROBLEM

*A. Introduction to collateral Portfolio and optimization*

A collateral portfolio is the set of all assets from organization inventory, pledged to all counterparties and the amount that can be borrowed. Optimization is the reallocation of collateral minimizing the cost of assets.

B. Cost Calculation Process

Given the client in each type of asset assumes different costs based on

| | Cash | | Security | | |
|---|---|---|---|---|---|
| Condition | Long (Client Pledged-in) | Short(Cash Borrowed) | MCR(Client Pledged-in) | Borrowed with Long Cash | Borrowed with short cash |
| Cost | Bid – max(0,yield+bp spread) | Ask – max(0,yield+bpspread) | Long Rank – Spread of Long rank | Bid - yield | Ask - Yield |

C. Portfolio Optimization LPP Model formulation

The Decision variables are the collateral allocations in nominals. Assuming m are the assets $A_1$, $A_2$, $A_3$,…, $A_m$that includethe borrowed onesand can be used to expose a set of n clients $C_1$, $C_2$, $C_3$,…, $C_n$. If asset $A_i$ is eligible for client $C_j$, then $x_{ij}$ is the decision variable [1] representing the nominal of asset $A_i$ allocated to client $C_j$.The total mn decision variables are illustrated by

$$\{x_{11},x_{12},\ldots,x_{21},x_{22},\ldots,x_{m1},x_{m2},\ldots,x_{mn}\}.$$

The Objective Function represents the entire cost of the portfolio, and our goal is to minimize it. Every asset may not be eligible for every client. Hence, the cost is a function of the asset and the client. If asset $A_i$ is eligible for client $C_j$, then the cost associated with this allocation per unit is $c_{ij}$. $P_i$ is the prince of the asset $A_i$, $hc_{ij}$ is the value of the haircut of the asset $A_i$For client $C_j$ and $fx_i$If FX is the ratioof the asset currency to the client call currency then

$c_{ij}(\mathbf{hc_{ij}}0.01*p_i*fx_i)*x_{ij}$ is the cost of allocating nominal $x_{ij}$ of asset $A_i$ to client $C_j$.

The Objective function is represented by

$$M = \text{Min } \sum c_{ij}(\mathbf{hc_{ij}}0.01*p_i*fx_i)*x_{ij} \quad 1\leq i \leq m,\ 1\leq j \leq n.$$

We need four types of constraints that satisfythe client exposure, the asset availability, the minimum filland nonnegativity.

i. Client exposure constraints

If a Client $C_j$ has exposure $e_j$, then the sum of the haircut values of the assets allocated to client $C_j$ must be equal to $e_j$

$$\sum(æ_{ij}*hc_{ij}*0.01*p_i*fx_i*x_{ij}) = e_j \qquad 1\leq i \leq m,\ 1\leq j \leq n$$

where $æ_{ij} = 1$ is asset $A_i$ is eligible for client $C_j$ otherwise 0

ii. Asset availability constraints

If the availability of an asset $A_i$ is $u_i$ (nominal) [2], then the sum of all nominal of asset $A_i$ allocated to eligible clients $C_j$ should be utmost $u_i$

$$\sum(æ_{ij} * x_{ij}) \leq u_i,\ 1\leq j \leq n\ 1\leq i \leq m$$

where $æ_{ij} = 1$ is asset $A_i$ is eligible for client $C_j$ otherwise 0

iii. Minimum Fill constraints

To minimize the number of substitutions nominal xij of asset Ai allocated to client Cj can be restricted to 100000 or minimum allocation from the existing portfolio.

iv. Nonnegativity constraints

$$x_{ij} \geq 0,\ 1\leq j \leq n\ 1\leq i \leq m$$

D. Solution

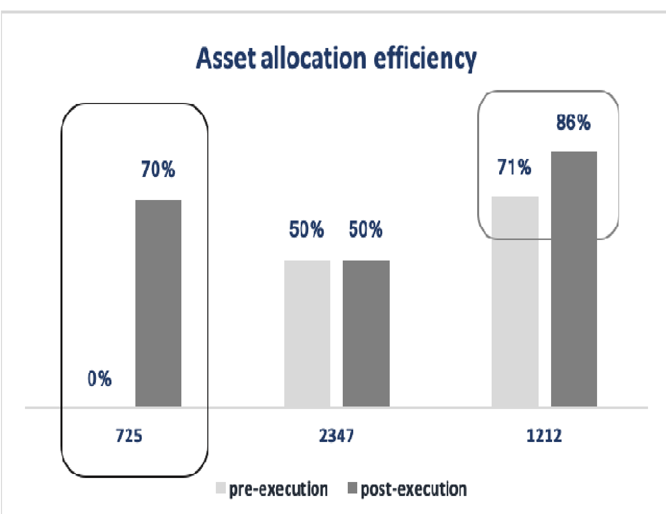
LPP problem formulated in section D is mixed integer linear programming[3] problem. This can be solved using Duel simplex method. There are a lot of software packages to solve this. I particularly used an opensourcelpsolve library which is implemented in C and has wrappers for Java, and R.

## III. RESULTS AND EXAMPLE

A project was implemented in an Investment bank using this methodology[4] which produced at least 50M dollars savings per annum. The results obtained demonstrated the methodology used as a sample portfolio.

| Portfolio before optimization | | | | |
|---|---|---|---|---|
| | Client ID | | | |
| Assets | 725 | 2347 | 1212 | Asset Availability (Nominal) |
| German Bond1 | 21,309,827 | X | X | 20,000,000 |
| Gilts1 | 17,182,848 | 0 | 0 | 10,000,000 |
| Gilts2 | 0 | 0 | 27,918,000 | 20,000,000 |
| French Bond1 | 0 | 36,579,573 | 0 | 30,000,000 |
| French Bond2 | 0 | 0 | 11,065,154 | 10,000,000 |
| French Bond3 | 0 | 0 | 0 | 20,000,000 |
| Italian Bond1 | X | 20,038,688 | X | 20,000,000 |
| US Bond1 | X | X | 9,207,000 | 10,000,000 |
| Exposure (USD) | 38,492,675 | 56,618,261 | 48,190,14 | |
| Weighted cost | 0.08 | 0.05 | 0.10 | |
| Efficiency | 0 | 50 | 71.42 | |
| Portfolio Cost | 10,413,711 | | | |

| Portfolio after optimization | | | | |
|---|---|---|---|---|
| | Client ID | | | |
| Assets | 725 | 2347 | 1212 | Asset Availability (Nominal) |
| German Bond1 | 11,514,765 | X | X | 20,000,000 |
| Gilts1 | 0 | 0 | 17,181,130 | 10,000,000 |
| Gilts2 | 0 | 0 | 27,918,000 | 20,000,000 |
| French Bond1 | 0 | 36,578,354 | 0 | 30,000,000 |
| French Bond2 | 7,975,763 | 0 | 3,088,284 | 10,000,000 |
| French Bond3 | 19,000,000 | 0 | 0 | 20,000,000 |
| Italian Bond1 | X | 20,038,688 | X | 20,000,000 |
| US Bond1 | X | X | 0 | 10,000,000 |
| Exposure (USD) | 38,492,675 | 56,618,261 | 48,190,14 | |
| Weighted cost | 0.06 | 0.05 | 0.08 | |
| Efficiency | 70 | 50 | 85 | |
| Portfolio Cost | 8,932,234 | | | |




## IV. CONCLUSION

The proposed method is well proven for all practical purposes and helps to reduce counterparty risk while optimizing asset usage. The example above shows 1M plus savings in the cost, and it saves many dollars that are vital for financial organizations.

## V. REFERENCES


[1] N.Mahdavi-Amiriab, S.H.Nasseria, “Duality results and a dual simplex method for linear programming problems with trapezoidal fuzzy variables” ELSEVIER, Volume 158, Issue 17, September 2007

[2] Benichou, M., Gauthier, J.M., Girodet, P. et al. Mathematical Programming (1971) 1: 76. https://doi.org/10.1007/BF01584074

[3] Michel Berkelaar,Samuel E. Buttrey, “lpSolve: Interface to 'Lp_solve' v. 5.5 to Solve Linear/Integer Programs” https://CRAN.R-project.org/package=lpSolve

[4] Tony J. Van Roy,Laurence A. Wolsey, “Solving Mixed Integer Programming Problems Using Automatic Reformulation”