

Robustness of PI-PD Controller Used With Third Order Processes

Akram F. Singer¹, Galal A. Hassaan², Mohamed A. Elgamil³

1(M.Sc. candidate Department of Mechanical Design & Production, Cairo University/ Faculty of Engineering, Cairo, Egypt
Email: akramfahmy40@gmail.com)

2 (Emeritus Prof. Department of Mechanical Design & Production, Cairo University/ Faculty of Engineering, Cairo, Egypt
Email: galalhassaan@ymail.com)

3(Assistant Prof. Department of Mechanical Design & Production, Cairo University/ Faculty of Engineering, Cairo, Egypt
Email: hamedeen@yahoo.com)

Abstract:

Robustness is one of the requisites used in controllers design. The objective of this research is to investigate the robustness of a PI-PD controller used to control a third-order process against uncertainty in the process parameters. A variation of $\pm 20\%$ in process parameters is considered through simulation to study its effect on the system performance parameters using the tuned controller. The variation of the process integral gain has small effect on the settling time, maximum percentage overshoot of the control system. The variation in process time constant T_1 resulted in a decrease of 12% in the maximum percentage undershoot, an increase of 34.2% in the maximum percentage overshoot and an increase of 34.2% in the settling time of the closed-loop control system. The variation in process time constant T_2 resulted in a decrease of 15.2% in the maximum percentage overshoot, a decrease of 6% in the maximum percentage undershoot and an increase of 5.9% in the settling time of the closed-loop control system. For all the changes in the process parameters the phase margin is from 64.4 to 66.2 (deg), and the gain margin is from 15 to 18.5 (dB).

Keywords — Third order process; PI-PD controller; uncertainty in process parameters; controller robustness; control

I. INTRODUCTION

During operation, processes are subject to uncertainty in their parameters. Therefore, it is important to investigate the effectiveness of the used controller in dealing with such uncertainty. Hu, Chang, Yeh and Kwatny (2000) used the H_∞ approximate I/O linearization formulation and μ -synthesis to design a nonlinear controller for an aircraft longitudinal flight control problem and address tracking, regulation and robustness issues [1]. Gong and Yao (2001) generalized a neural network adaptive robust control design to synthesize performance oriented control laws for a class of nonlinear systems in semi-strict feedback forms through the incorporation of back stepping design techniques [2]. Lee and Na (2002) designed a robust controller for a nuclear power control system. They used the Kharitonov and edge theorem to determine the controller which was

simpler than that obtained by the H_∞ [3]. Arvanitis, Syrkos, Stellas and Sigrimis (2003) analyzed PDF controllers designed and tuned to control integrator plus dead time processes in terms of robustness. They performed the robustness analysis in terms of structured parametric uncertainty description [4]. Lhommeau, Hardouin, Cottenceau and Laulin (2004) discussed the existence and the computation of a robust controller set for uncertain systems described by parametric models with unknown parameters assumed to vary between known bounds [5]. Dechanupaprittha, Hongesombut, Watanabe, Mitani and Ngammroo (2005) introduced the design of robust superconducting magnetic energy storage controller in a multi machine power system by using hybrid tabu search and evolutionary programming. The objective function of the optimization problem considered the disturbance attenuation performance and robust stability index [6]. Chin, Lau, Low and Seet (2006) proposed a robust PID controller based on actuated

dynamics and an un actuated dynamics shown to be global bounded by the Sordalen lemma giving the necessary sufficient condition to guarantee the global asymptotic stability of the URV system [7]. Vagja and Tzes (2007) introduced a robust PID controller coupled into a Feed forward compensator for set point regulation of an electrostatic micromechanical actuator. They tuned the PID controller using the LMI-approach for robustness against the switching nature of the linearized system dynamics [8]. Fiorentini and Bolender (2008) described the design of a nonlinear robust/adaptive controller for an air-breathing hypersonic vehicle model. They adapted a nonlinear sequential loop-closure approach to design a dynamic state-feedback control for stable tracking of velocity and altitude reference trajectories [9]. Labibi, Marquez and Chen (2009) presented a scheme to design decentralized robust PI controllers for uncertain LTI multi-variable systems. They obtained sufficient conditions for closed-loop stability of multi-variable systems and robust performance of the overall system [10]. Matusu, Vanekova, Porkop and Bakosova (2010) presented a possible approach to design simple PI robust controllers and demonstrate their applicability during control of a laboratory model with uncertain parameters through PLC [11]. Kada and Ghazzawi (2011) described the structures and design of a robust PID controller for higher order systems. They introduced a design scheme combining deadbeat response, robust control and model reduction techniques to enhance the performance and robustness of the PID controller [12]. Surjan (2012) applied the genetic algorithm for the design of the structure specified optimal robust controllers. The parameters of the chosen controller were obtained by solving the nonlinear constrained optimization problem using IAE, ISE, ITAE and ITSE performance indices. He used constraints on the frequency domain performances with robust stability and disturbance rejection [13]. Jiao, Jin and Wang (2013) analyzed the robustness of a double PID controller for a missile system by changing the aerodynamic coefficients. They viewed the dynamic characteristics as a two-loop system and designed an adaptive PID control strategy for the pitch channel linear model of supersonic missile [14].

Pradhama, Ray, Sahu and Moharana (2014) proposed a control strategy to improve the power factor and voltage regulation at a distribution supply system for more robustness [15]. Hassaan (2014) studied the robustness of a feedback PD compensator used with both second-order and third-order processes. He showed that this compensator is completely robust for process parameters variation in the range $\pm 20\%$ [16]. Emma D. Welson et al (2018), introduced that evaluation of closed-loop robustness has generally relied on empirical methods. They have proved that, expressions for the $H_\infty H_\infty$ norm of two commonly used PIP control implementations, the feedback and forward path forms, are used, for the first time, to quantify closed-loop robustness [17]. Bharat Verma and Prabin Kumar Padhy (2019), focused on online PID controller tuning with the guaranteed robustness of the controller. A new single variable tuning method is developed for the online robustness and performance adjustment. They implemented that, the proposed rules only depend upon the previously optimized PID parameters.[18]. Min Zheng, Tao Huang and Guangfeng Zhang (2019), proposed that robust tuning of controller parameter is considered an effective way to deal with continuously changing end-user specs and raw product properties. They showed that, the specifications such as settling time, overshoot and robustness have a direct meaning in terms of process output and remain most popular amongst process engineers. They implemented an intuitive tuning procedure for robustness which is based on linear system tools such as frequency response and band limited specifications thereof, loop shaping remains a mature and easy to use methodology [19]. Clara M. Ionesco et al (2020), showed that successful operation in a globalization context can only be ensured by robust tuning of controller parameter as an effective way to deal with continuously changing end-user specs and raw product properties. They presented that; Recently next to these popular loop shaping methods, new tools have emerged, i.e. fractional order controller tuning rules. The key feature of the latter group is an intrinsic robustness to variations in the gain, time delay and time constant values, hence ideally suited for loop shaping purpose. They sketched and

discussed both methods in terms of their advantages and disadvantages [20].

II. PROPOSED ALGORITHM

A. The Process

The process considered in this analysis is a third order process having the following forward transfer function in a unity feedback system as shown in Fig.1:

$$G_p(s) = [K_{ip}\omega_n^2 / (s^3 + 2\zeta\omega_n s^2 + \omega_n^2 s + K_{ip}\omega_n^2)] \quad (1)$$

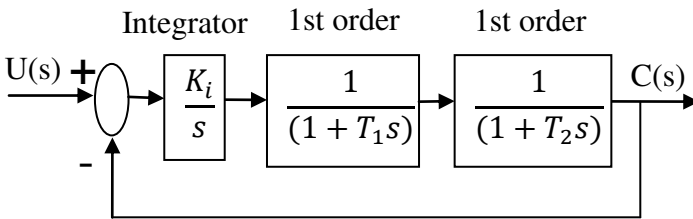


Fig. 1 Block diagram of third order process simulator

where

- Kip ... integral gain of the process (in this prescribed third order process Kip = 0.5)
- ω_n ... natural frequency ($\omega_n = 0.447$ rad/s)
- ζ ... damping ratio ($\zeta = 1.34$)
- T1 ... Time constant (T1 = 1s)
- T2 ... Time constant (T2 = 5s)

B. The PI-PD Controller

A [proportional + integral] (PI) - [proportional + derivative] (PD) controller type is used in this research. The parts of the controller used in this study are connected in series. The input to the PD part is the output of the controlled system, and the PI controller part is connected in series. The output of the PD part is subtracted from the second summing point as shown in Fig.2. [23,24]. The output signal of the second summing point is the control signal acting on the controlled third order process.

C. Control System Transfer Function

The process output C(s), is related to its input U(s) through the process transfer function, $G_p(s)$. That is:

$$C(s) = G_p(s) U(s)$$

this control system has the transfer function M(s) [21]:

$$M(s) = [b_0 s^4 + b_1 s^3 + b_2 s^2 + b_3 s + b_4] / [a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4] \quad (2)$$

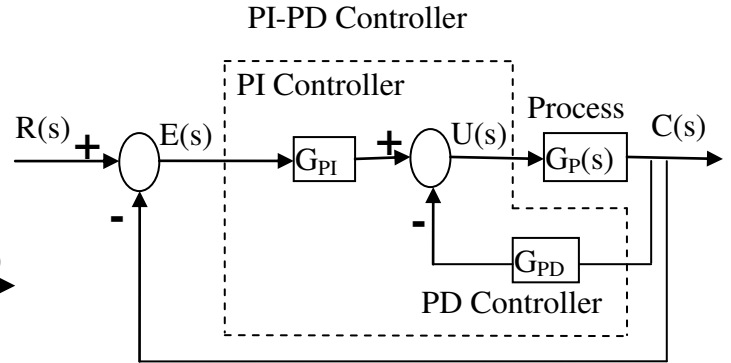


Fig. 2 PI-PD controller-based control system [23]

Where

$$b_0=0, b_1=0, b_2=0, b_3=(K_{ip}K_{pc1}\omega_n^2), b_4=(K_{ip}K_i\omega_n^2). \\ a_0=1, a_1=(2\zeta\omega_n), a_2=(\omega_n^2 + K_d K_{ip}\omega_n^2), a_3=(K_{pc1}K_{ip}\omega_n^2 + K_{ip}K_{pc2}\omega_n^2), a_4 = (K_i K_{ip}\omega_n^2).$$

- (K_i) Integral gain of the PI controller
 - (K_{ip}) Integral gain of the prescribed third order process ($K_{ip} = 0.5$)
 - (K_d) Derivative gain of the PD controller
 - (K_{pc1}) Proportional gain of the PD controller
 - (K_{pc2}) Proportional gain of the PI controller
- The controller has four parameters K_i, K_d, K_{pc1} , and K_{pc2} .

D. Controller Tuning

The PI-PD controller was tuned by the author to control this third order process [25]. The controller parameters are tuned as follows:

- Control and optimization toolboxes of MATLAB are used to assign the four parameters of the controller (K_i, K_d, K_{pc1} , and K_{pc2}). Their values are [25]:

$$K_{pc1} = 0.0713, K_{pc2} = 5.83, K_i = 0.93, K_d = 15.35.$$

E. Process Uncertainty

Due to the variation in the operation conditions during operation, the process is submitted to parametric changes. It is supposed that this

change be as large as $\pm 20\%$ of the assigned process parameters.

III. CONTROLLER ROBUSTNESS

The control system considered robust if it has acceptable changes in its performance due to model changes or inaccuracy [28]. Furthermore, Lee and Na add the stability requirement to the robustness definition besides the plants having uncertainty [3]. Toscano adds that the controller has to be able to stabilize the control system for all the operating conditions [21]. In this research, the robustness of the controller and hence of the whole control system is assessed as follows:

- Nominal process parameters are identified.
- The controller is tuned for those process parameters.
- A variation of the process parameters is assumed within a certain range.
- Using the same controller parameters, the step response of the system using the new process parameters is drawn and the control system performance is evaluated through the maximum percentage overshoot, maximum percentage undershoot and settling time.
- The frequency based relative stability parameters are also evaluated using the open-loop transfer function of the control system.
- The variation in process parameters is increased and the procedure is repeated.

The effect of the variation of process parameters on the settling time, maximum percentage overshoot, maximum percentage undershoot, gain margin and phase margin of the closed loop control system using the tuned PI-PD controller parameters are shown in Figs.3, 4,5, 6 and 7.

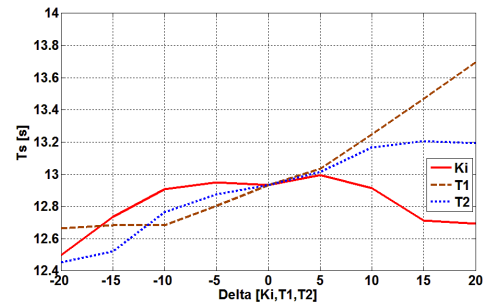


Fig. 3 Effect of process parameters change on system settling time

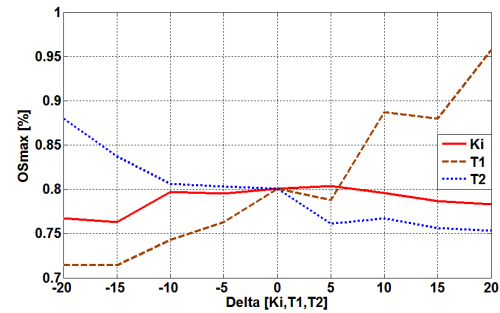


Fig. 4 Effect of process parameters change on system maximum percentage overshoot

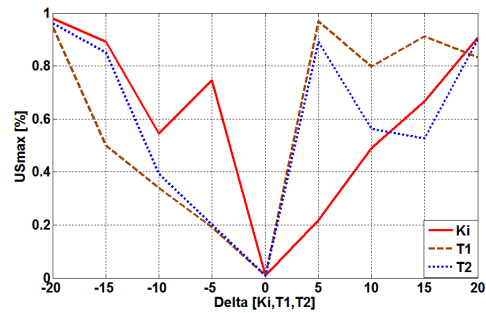


Fig. 5 Effect of process parameters change on system maximum percentage undershoot

According to OGATA [29], for a control system with good performance:

- Gain margin: has to be > 6 dB.
- Phase margin: has to be in the range: $30 \leq PM \leq 60$ degrees.

According to Lei and Man [30], the phase margin range can be widened to be:

$$30 \leq PM \leq 90$$

The open loop transfer function of the closed loop control system incorporating the PI-PD controller and the third order process, using the block diagram of Fig.2, is:

$$G(s)H(s) = [(k_i \cdot k_{pc1})s + k_i^2] / [(T_1 T_2)s^4 + (T_1 + T_2)s^3 + (k_i k_d + 1)s^2 + (k_i k_{pc2})s] \quad (3)$$

Using the open loop transfer function of Eq.3 and the command 'margin' of the MATLAB program, the Gain Margin and Phase Margin of the control system against the variations in the process parameters are shown in Figs.6 and 7.

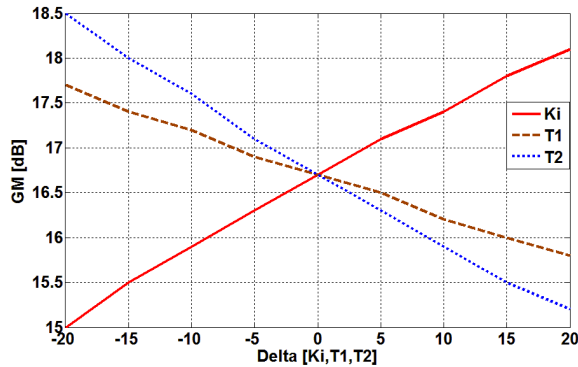


Fig. 6 Effect of process parameters change on system gain margin (GM)

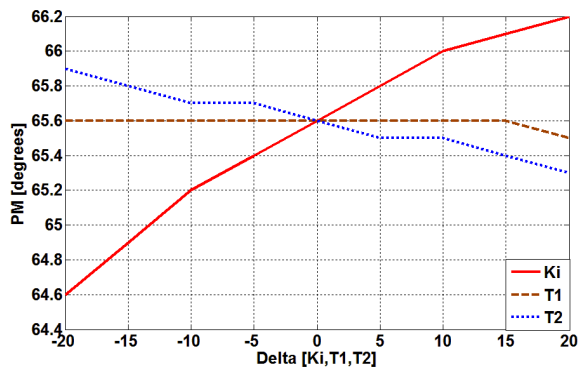


Fig. 7 Effect of process parameters change on system phase margin (PM)

IV. CONCLUSIONS

- The Variation in third order process parameters within $\pm 20\%$ was considered.
- Tuned PI-PD controller is robust since it controlled the third order process for set-point change maintaining good performance and stable control system for the range of parameters change.
- With the PI-PD controller, a change of 20% in process integral gain K_{ip} resulted in an increase in both the settling time by 1.5% and the maximum percentage overshoot by 2%, but it resulted in a decrease in the maximum percentage undershoot by 7.3% .

- With the PI-PD controller, a change of 20% in process time constant T_1 resulted in an increase in both the settling time by 8.2% and the maximum percentage overshoot by 34.2%, but it resulted in a decrease in the maximum percentage undershoot by 12%.
- With the PI-PD controller, a change of 20% in process time constant T_2 resulted in an increase the settling time of the closed-loop control system by 5.9%, and a decrease in both the maximum percentage overshoot by 15.2%, and the maximum percentage undershoot by 6%.
- The gain margin is sensitive to the changes in the parameters of the third order process.
- The phase margin is not sensitive with the changes in the time constant T_1 of the third order process.
- The phase margin of the control system is within the range assigned by Lee and Man [30].

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