A NEW NUMERICAL APPROACH INVESTIGATION FOR BUCKLING AND VIBRATION ANALYSIS OF CRACKED COLUMNS

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Abstract- In this paper numerical plan for buckling investigation of a section and vibration examination of a shaft is introduced. The shaft and segment is thought to be non-uniform and cracked. Utilizing analytics of varieties the issue is communicated as an enhancement issue. A procedure of improvement is utilized for investigation of buckling load. Thinking about the similitude between the overseeing condition for buckling and free vibration, the crucial recurrence and mode state of the shaft is registered with a similar technique. A few models are settled. The outcomes are contrasted and different techniques. A fantastic understanding was gotten.

Keywords: - Buckling, vibration, cracked, non-uniform, beam, column

I. Introduction

The presence of a crack in a structural member presents a nearby adaptability that influences its dynamic reaction. The progressions in powerful qualities can be estimated and lead to a recognizable proof of structural changes, which in the end may prompt the identification of a structural blemish. An abundance of logical, mathematical and exploratory examinations presently exists. References more identified with the current investigation are refered to here.

T.G. Chondros, et al. [1] fostered a nonstop cracked pillar vibration hypothesis for the parallel vibration of cracked Euler-Bernoulli radiates with single-edge or twofold edge open cracks. They utilized the Hu-Washizu-Bar variational definition to foster the differential condition and the limit states of the cracked shaft as a one-dimensional continuum. The uprooting field about the crack was utilized to adjust the pressure and relocation field all through the bar. A steel bar with a twofold edge crack was researched and results contrasted well and existing test information. They expanded their hypothesis for a bar with breathing crack [2].

E.I. Shifrin, et al. [3] proposed a method for figuring normal frequencies of a vibrating bar with subjective number of cross over open cracks. The primary element of their strategy is identified with diminishing the element of the framework engaged with the estimation, so that decreased calculation time is needed for assessing normal frequencies contrasted with elective techniques.

Vibration of shafts with numerous open cracks exposed to pivotal power is concentrated by Baric Binici [4]. He proposed a technique to get the eigenfrequencies and mode states of pillars containing various cracks and exposed to pivotal power. The technique utilizes one bunch of end conditions at crack areas; mode shape elements of residual parts are resolved. Other arrangement of limit conditions yields a second-request determinant that should be addressed for its underlying foundations. He thought about both vibration and buckling heap of the structure.

M. Behzad, et al. [5] dependent on Hamilton standard fostered the condition of movement and relating limit conditions for twisting vibration of a pillar with an open edge crack. The normal frequencies of a uniform Euler-Bernoulli bar have been determined utilizing the new evolved model related to the Galerkin projection strategy. Buckling of multi-step crack segments with shear misshapening is concentrated by Q. S. Li [6]. The administering differential condition for buckling of one-venture cracked section with shear twisting is set up and its answer is discovered first. Then, at that point another methodology that consolidates the specific buckling arrangement of a one-venture section and the exchange lattice strategy (TMM) is introduced for addressing the whole and fractional buckling of a multi-step segment with different end conditions, with or without cracks and shear misshapening, exposed to concentrated hub load. The fundamental benefit of the proposed strategy is that the eigen-esteem condition for buckling of a multi-step segment with a discretionary number of cracks, any sort of two end upholds and different spring upholds at middle focuses can be advantageously resolved from an arrangement of two direct conditions. He later [7], broadened the technique and proposed classes of precise answers for buckling of multi-step non-uniform segments with a discretionary number of cracks exposed to concentrated and disseminated pivotal burdens.

Ranjbaran A. proposed a strategy for calculation of principal eigen-sets of shear building [8]. The major mode was acquired as least of a remainder characterized as far as strain energy and snapshot of idleness as a component of sidelong relocation. The mode shape is acquired as minimizer uprooting capacity of the remainder.

In the current paper and in a large portion of references refered to, the crack is demonstrated as a rotational spring with indicated adaptability [9]. In view of analytics of varieties, the issue of calculation of buckling heap of segments and vibration of bars, are characterized as a streamlining of a remainder. The minimizer capacity of the remainder is the mode shape and its base worth is buckling burden or key recurrence of the bar. When contrasted with others the proposed strategy is more basic, proficient and precise.

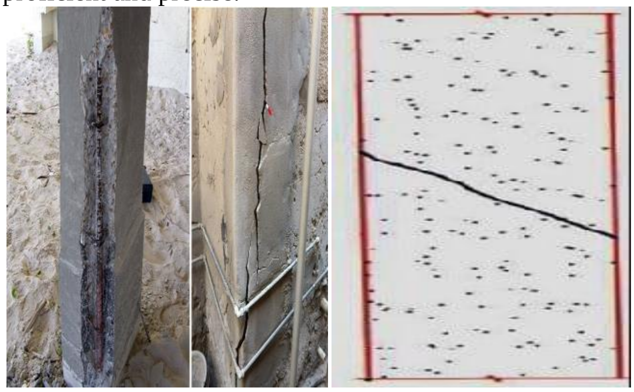

Figure-1 Cracked Column

2.THEORETICAL BASIS

2.1 CRACK MODEL

A crack in a column is modeled as a rotational spring with flexibility coefficient, C, defined as [9]:

$$C = 5.346Hf(\xi), \qquad \xi = \frac{a}{H}$$

In which H is the column section height and a is the depth of the crack. The function f is defined as:

$f(\xi) = 1.862\xi^2 - 3.95\xi^3 + 16.375\xi^4 - 37.22\xi^5 + 76.81\xi^6 - 126\xi^7 + 172\xi^8 - 143.97\xi^9 + 66.56\xi^{10}$……………..(2)

The spring inserts a jump in the rotation of column centerline at the cracked point, i.e. as:

$$\frac{d\theta}{dx} = \frac{d\emptyset}{dx}\frac{d^2y}{dx^2}, \quad OR \quad \Delta\theta = C\frac{d^2y}{dx^2} = C\frac{M}{EI}, \quad C = d\emptyset$$

Where θ is rotation and φ is a potential function.

2.2 BUCKLING OF A COLUMN

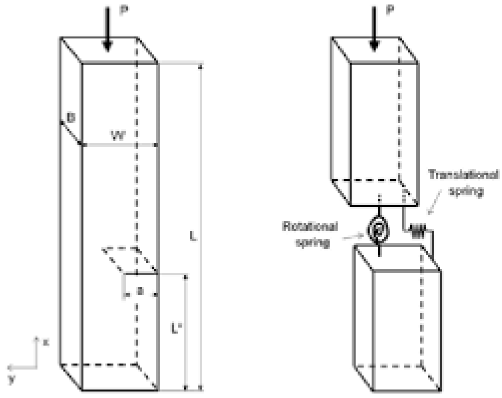

Figure-2

The equilibrium equation for a non-uniform cracked column in a displaced position is expressed as:

$$L^2y^{\parallel} + qry = 0, \quad y(0) = y(L) = 0$$

In which prime (/) denote differentiation with respect to x and:

$$q = \frac{PL^2}{EI_c}, \qquad r = \left(1 + \frac{d\emptyset}{dx}\right)\frac{EI_c}{EI}$$

Where E is elastic modulus, I is second moment of area, L is length, q is a working parameter and c in the subscript denotes a specified section. The ratio r is selected for modeling non-uniform sections.

A functional in form of a quotient is defined as follows:

$$q = \frac{L^2\int_0^L y^2dx}{\int_0^L ry^2dx} = \frac{\int_0^L f_1dx}{\int_0^L f_2dx}$$

In which f_1 and f_2 are selected for better reference. According to calculus of variations, the Euler-Lasgrange equation corresponding to equation (6) is written as:

$$F = -(f_{1y} - q_0f_{2y}) + \frac{d}{dx}\,(f_{1y} - q_0f_{2y}) = 0$$

In which the letter in the subscript denotes differentiation and q_0 is minimum value of the quotient q. Substitution from equation (6) into equation (7) leads to the following equation:

$$F = -(0 - 2qry) + \frac{d}{dx}(2L^2y - 0) = 0 \rightarrow L^2y + qry = 0$$

Equation (8) clearly shows that the minimizer of quotient q is the displacement function y that satisfies the governing equation and corresponding boundary conditions. As a result the problem of computation of buckling load of the columns may be expressed as the following minimization problem:

$$Minimize : q = \frac{L^2\int_0^L y^2dx}{\int_0^L ry^2dx}$$
$$Subject\ to : y(0) = y(L) = 0$$

For numerical computation, the column is divided into n segments connected at n+1 nodes. Finite difference scheme is used in place of derivatives. The quotient q may be expressed in terms of nodal values as follows:

$$q = \frac{L^2 \sum_{\alpha=1}^{n-1} y_\alpha (2y_\alpha - y_{\alpha-1} - y_{\alpha+1})}{h^2 \sum_{\alpha=1}^{n-1} r_\alpha y_\alpha^2 dx}, y_0 = y_n = 0,\ h = \frac{L}{n}$$

The BFGS algorithm is used for carrying out the minimization process of quotient q in equation (10) [10].

After finding the quotient q, the critical load P, ratio of critical load, P, to Euler load, Pe, i.e. R_P, and effective length factor, K, is computed as follows:

$$P = q\frac{EI_c}{L^2},\ \ R_P = \frac{P}{P} = \frac{q}{\pi^2}, \qquad K = \frac{1}{\sqrt{R_P}} = \frac{\pi}{\sqrt{q}}$$

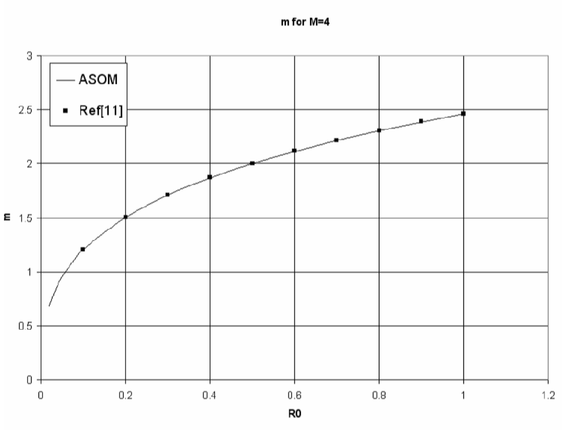

Figure-3 Variation of m versus R0 in example 1

Example 1

In order to verify the formulation for non-uniform column a simply supported column with variable cross section is selected for analysis. The following parameters are used:

$$I_x = I_0 \left(\frac{x+a}{a}\right)^M$$

In which I_0, I_c, and Ix are second moment of area at sections o, c, and x respectively and M is a power number.
Solution

For this problem the ratio r_i may be shown as:

$$r_i = \left[\sqrt[M]{R_0} + 2\left(1 - \sqrt[M]{R_0}\right)\left(\frac{x_i}{L}\right)\right]^{-M},\ \ R_0 = \frac{I_0}{I_c}$$

Note that because of symmetry for half-length of the column ri is computed as above and for the other half symmetry is used. The critical load of the column is denoted as:

$$P_{cr} = \frac{4mEI_c}{L^2}$$

This problem is solved in reference [11] for M=2 and 4. The problem is solved for R0 = 0.1, 0.2, .., 1.0 and M= 4. The results of the present study and that of reference [11] are compared in Figure1. Excellent agreement of results obtained.

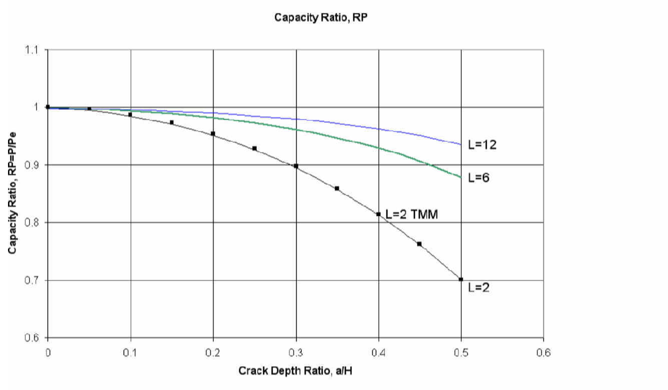

Figure -4 RP versus a/H and L

Example 2

A simply supported column of length L, crack depth a, and crack position L L with a rectangular cross section of 20 cm height and 15 cm width is considered. The elastic modulus is 200Gpa. The buckling load is computed and its variation versus crack depth Ratio for different length (slenderness ratio) is shown in Figure 2. The crack is at mid-height of the column. For L=2 the results of Transformation Matrix Method (TMM) is also shown for comparison.

2.3 VIBRATION ANALYSIS OF A BEAM

The governing equation for free vibration of a beam is similar to that of buckling load of a column except in the name of parameters. Based on this similarity the circular frequency may be obtained by using $P \approx \rho A \left(L/\pi\right)^2 \omega^2$ where ρ is mass density, A is cross sectional area and ω is circular frequency. As a result the circular frequency of a beam is obtained as:

$$\omega^2 = \frac{q}{\pi^2}\left(\frac{\pi}{L}\right)^4 \frac{EI_c}{\rho A_c}, \qquad \omega_c^2 = \left(\frac{\pi}{L}\right)^4 \frac{EI_c}{\rho A_c}$$

And the ratio of ω^2 to ω^2, i.e. Rω is defined as:

$$R_\omega = \frac{\omega}{\omega_c} = \frac{\sqrt{q}}{\pi} = \sqrt{R_P}$$

In case of uniform members no other consideration is needed.
For the case of non-uniform column the ratio r should be defined as follows:

$$r = \left(1 + \frac{d\emptyset}{dx}\right)\frac{EI_c}{EI}\frac{A}{A_c} = \left(1 + \frac{d\emptyset}{dx}\right)\frac{Er_{gc}^2}{Er_g^2}, \qquad r_g^2 = \frac{I}{A}$$

Example 3
The same member as in example 2 is considered but now as a beam. The variation of frequency ratio versus crack depth ratio for different lengths is shown in Figure 3.

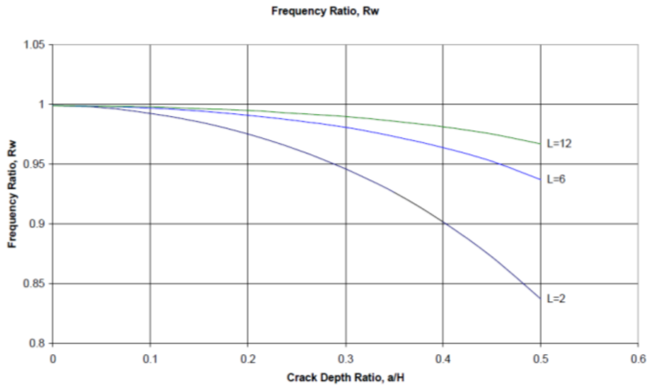

Figure -5 RW versus a/H and L

3. CONCLUSION

A new, efficient and accurate method for vibration analysis of a non-uniform cracked beam is presented. Based on the calculus of variations the theoretical basis of the method is developed. First computation method for analysis of buckling load of a non-uniform column is presented. Considering the similarity between the governing equations of buckling analysis of columns and free vibration analysis of beam the latter was formulated. To verify the proposed method several examples are presented. Results of this study are compared with that of others. An excellent agreement was obtained. As compared to others, the volume of formulations, the amount of computer algorithm needs, and the time of computations, is considerably less. At the same time the presented method is quite general.

REFRENCES

[1] T. G. Chondoros, A. D. Dimarogonas, and J.Yao "A Continuous Cracked Beam
Vibration Theory" *Journal of Sound and Vibration* (1998) 215(1), 17-34.
[2] T. G. Chondoros, A. D. Dimarogonas, and J.Yao "Vibration of a Beam with
Breathing Crack" *Journal of Sound and Vibration* (2001) 239(1), 57-67.
[3] E.I. Shifrin and R. Ruotolo "Natural Frequencies of A Beam With an Arbitrary
Number of Cracks" *Journal of Sound and Vibration* (1999) 222(3), 409-423.
[4] Bris Bicini "Vibration of Beams with Multiple Open Cracks Subjected to Axial
Force" *Journal of Sound and Vibration* 287(2005), 409-423.
[5] M. Behzad, A. Meghdari, and A. Ebrahimi "A New Approach for Vibration
Analysis of A Cracked Beam" *International Journal of Engineering* (2005), 18(4) 319-
330.
[6] Q. S. Li "Buckling of Multi-Step Cracked Columns with Shear Deformation"
Engineering Structures 23(2001), 356-364.
[7] Q. S. Li "Classes of Exact Solutions for Buckling of Multi-Step Non-Uniform
Columns with An Arbitrary Number of Cracks Subjected to Concentrated and
Distributed Axial Loads" *International Journal of Engineering Science* 41(2003), 569-
586.
[8] A.Ranjbaran "ASOM Applied to Dynamic Analysis" *Communication In Numerical
Methods in Engineering*, 22(9), September 2006, pp. 967-074.
[9] Dimarogonas A.D. "Vibration of Cracked Structures A State of The Art Review"
Engineering Fracture Mechanics (1996), 55(5), 831-850.
[10] R. W. Pike "*Optimization of Engineering Systems"* Von Nostrand Reinhold New
York, 1985.
[11] S. P. Timoshenko and J. M. Gere *"Theory of Elastic Stability*" McGraw-Hill Book
Company, 1963.