

Analytical Study of Vibration and buckling of Cracked Structural Members: Beam and Column

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Abstract:

An important task for engineers is to determine the effect of the damage like transverse cracks on the stability characteristic of structures. The presence of cracks causes changes in the physical properties of a structure and its dynamic response characteristics. The monitoring of the changes in the response parameters of a structure has been widely used for the assessment of structural integrity, performance and safety. The buckling load is one of the important parameter for stability of a structural member. The present work is aimed at finding the buckling load of a cracked beam-column with a single edge crack. Finite element method is adopted for the dynamic analysis of the beam-column. Additional flexibility coefficients of the cracked beam element are computed using 6- point Gauss quadrature and theories of fracture mechanics. Flexibility coefficients of an intact element are added to the additional flexibility matrix to get the total flexibility matrix of the element.

Key Words: Free Vibrations, Frequencies, Structural Cracks, Buckling, Beam-column.

1. INTRODUCTION

The cracks in a structural member may develop from flaws due to applied cyclic loads, mechanical vibrations, aerodynamic loads, rocket fuel exhaust or acoustical fatigue. In civil engineering, structures like beam columns, bridges, piles, etc will bear damages due to long- term service, collision, impact, etc. An important task of engineers is to determine the effect of these damages on the stability characteristic of these structures. The presence of cracks causes changes in the physical properties of a structure which in turn alter its dynamic response characteristics. The monitoring of the changes in the response parameters of a structure has been widely used for the assessment of structural integrity, performance and safety. Irregular variations in the measured

vibration response characteristics have been observed depending upon whether the crack is closed, open or breathing during vibration, the degree of severity and modal type. These variables consequently affect the effectiveness of structural integrity assessment. Members that are subjected to both bending and axial compression are beam-columns. Bending is caused by either moments applied to the ends of the member or it may be due to transverse loads directly acting on the member. Extensive studies have been done on the free vibration analysis of cracked beams and elastic stability of un-cracked columns. However, vibration and buckling analysis of a cracked beam-column have been studied only by a few researchers. The study of vibration and buckling load of a slender beam-column with crack is a problem of practical interest and finds applications in aerospace, mechanical and civil

engineering. Cracks or other defects in a structural element influence its dynamical behavior and change its stiffness and damping properties. Consequently, the natural frequencies and mode shapes of the structure contain information about the location and dimensions of the damage. Vibration analysis can be used to detect structural defects such as cracks, of any structure offer an effective, inexpensive and fast means of nondestructive testing. What types of changes occur in the vibration characteristics, how these changes can be detected and how the condition of the structure is interpreted has been the topic of several research studies in the past. The use of composite materials in various construction elements has increased substantially over the past few year.



Figure-1 Crack of Beam



Figure-2 Crack of Column

2. CRACK THEORY

Physical parameters affecting dynamic characteristics of cracked structures:

Usually the physical dimensions, boundary conditions, the material properties of the structure play important role for the determination of its

dynamic response. Their vibrations cause changes in dynamic characteristics of structures. In addition to this presence of a crack in structures modifies its dynamic behaviour. The following aspects of the crack greatly influence the dynamic response of the structure.

- I. The position of crack
- II. The depth of crack
- III. The orientation of crack
- IV. The number of cracks

CLASSIFICATION OF CRACKS

- Based on their geometries, cracks can be broadly classified as follows:
- Cracks perpendicular to the beam axis are known as “transverse cracks”. These are the most common and most serious as they reduce the cross-section and thereby weaken the beam. They introduce a local flexibility in the stiffness of the beam due to strain energy concentration in the vicinity of the crack tip.
- Cracks parallel to the beam axis are known as “longitudinal cracks”. They are not that common but they pose danger when the tensile load is applied is at right angles to the crack direction i.e. perpendicular to beam axis or the perpendicular to crack.
- “Slant cracks” (cracks at an angle to the beam axis) are also encountered, but are not very common. These influence the torsion behavior of the beam. Their effect on lateral vibrations is less than that of transverse cracks of comparable severity.
- Cracks that open when the affected part of the material is subjected to tensile stresses and close when the stress is reversed are known as “breathing cracks”. The stiffness of the component is most influenced when under tension. The breathing of the crack results in non-linearity’s in the vibration behavior of the beam. Cracks breathe when crack sizes are small, running speeds are low

and radial forces are large. Most theoretical research efforts are concentrated on “transverse breathing” cracks due to their direct practical relevance.

- Cracks that always remain open are known as “gaping cracks”. They are more correctly called “notches”. Gaping cracks are easy to mimic in a laboratory environment and hence most experimental work is focused on this particular crack type.
- Cracks that open on the surface are called “surface cracks”. They can normally be detected by techniques such as dye-penetrates or visual inspection.
- Cracks that do not show on the surface are called “subsurface cracks”. Special techniques such as ultrasonic, magnetic particle, radiography or shaft voltage drop are needed to detect them. Surface cracks have a greater effect than subsurface cracks on the vibration behavior of shafts

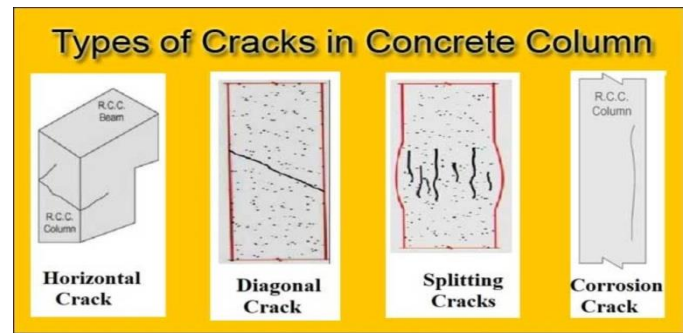


Figure-4 Types of the cracks

3- METHODOLOGY FINITE ELEMENT METHOD

The linear stability problem is accommodated in the FEM by the introduction of what is known as geometric stiffness which account for in plane loading on conventional bending stiffness, the effective stiffness vanishes at buckling load. This is an Eigen value problem with Eigen values now being the critical values of loading magnitude at which buckling occurs usually lowest of these is of p_r . The second type of problem comes from the realm of structural dynamics. Alternative is restricted to the calculation of common structural components and forms. This requires the development of mass matrix which will represent the effect of dynamic loading (proportional to the square of frequency) which is set up during vibration. In common with Eigen values now represent the square of the natural frequency and Eigen vectors defining the deformed shape of the structure when vibrating at a particular natural frequency.

The equation of motion in matrix form for vibration of a beam under load is written as

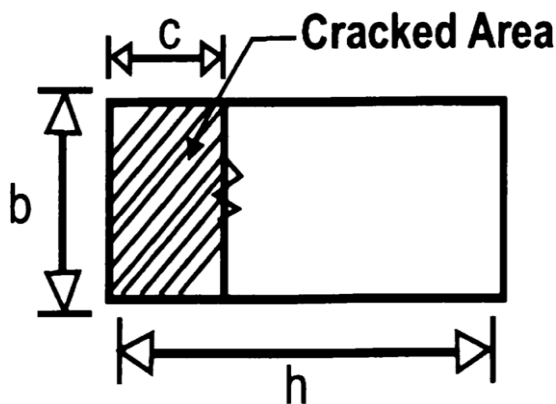
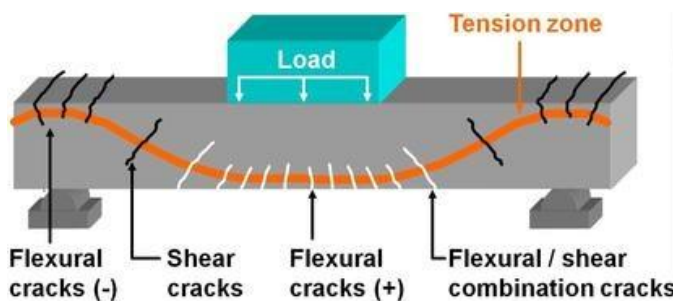


Figure-5 Cracked Area



$$[M]\{\ddot{q}\} + [K] - P[K_g]\{q\} = 0 \text{-----(1)}$$

Where,

$[M]$ = Consistent mass matrix

$[K]$ = Bending stiffness matrix of the beam

$[K_g]$ = Geometric stiffness matrix

$\{q\}$ = Displacement vector

P = External force vector

For free vibration the equation (1) can be written as,

$$[M]\{\ddot{q}\} + [K]\{q\} = 0 \text{----- (2)}$$

Where, the forcing function, $P = 0$

The equation (2) represents an Eigen value problem and the roots of the equation give rise to square of the natural frequency given by the equation,

The equation of motion in matrix form for buckling of a beam under load is written as

$$[K] - P_{cr}[K_g] = 0 \text{----- (3)}$$

The Finite Element Method (FEM) is a procedure for the numerical solution of the equations that govern the problems found in nature. Usually the behavior of nature can be described by equations expressed in differential or integral form. For this reason the FEM is understood in mathematical circles as a numerical technique for solving partial differential or integral equations. Generally, the FEM allows users to obtain the evolution in space and/or time of one or more variables representing the behavior of a physical system. When referred to the analysis of structures the FEM is a powerful method for computing the displacements, stresses and strains in a structure under a set of loads.

4-ADVANTAGES

- Irregular Boundaries
- General Loads
- Different Materials Boundary Conditions
- Variable Element Size
- Easy Modification

- Dynamics Nonlinear Problems (Geometric or Material)

5-CONCLUSIONS

- A method for identifying the crack location and depth of the uniform beam was developed by using the linear fracture mechanics theory. The finite element model of the cracked beam is constructed and used to determine its natural frequencies.
- Standard FEM procedure is followed which will lead to a generalized eigen value problem and thus natural frequencies, critical buckling loads are obtained.
- Stability study of the cracked beams can be done with three degrees of freedom per node.
- Shear deformation can be taken into consideration for the analysis.

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