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Estimation of Unknown Input of a System Using Full Order Observer

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Abstract:

This paper presents a simple unknown input estimation technique using generalized matrix inverse. The full order observer constructed by g-inverse is extended and implemented for this purpose. Full order observer constructed using g-inverse has been extended for the design of an unknown input observer and the unknown input has been estimated successfully from the observed states. The necessary conditions for the proposed estimation method are given in details. The proposed method is illustrated by numerical example (Two loop autopilot in pitch plane) and the MATLAB simulation results.

***Keywords* — Unknown Input Observer (UIO), Unknown Input Estimation (UIE), Generalized Matrix Inverse (or g-inverse), Full Order Observer, Unknown Input, Missile Autopilot.**

I. INTRODUCTION

This paper presents a simple unknown input estimation method of a linear time invariant system using full order observer. The theory of generalized matrix inverse has been used for derivation purpose. The necessary conditions for the proposed estimation method are given in details. A numerical example of a class of flight path rate demand missile autopilot (two loops) has been used to illustrate the proposed estimation scheme.

D. G. Luenberger first designed an observer in 1971 which approximately reconstructed missing state variable information [3]. Prof. Das and Prof. T. K. Ghoshal jointly proposed the method of construction of reduced order observer by using generalized matrix inverse [4]. Now to estimate the unknown input, it is necessary to design an unknown input observer (UIO). In [5], a constructive solution to the problem of designing a reduced order Luenberger observer for linear systems subject to arbitrary unknown inputs has been presented. A direct full-order observer design procedure for a linear system with unknown inputs is presented in [6]. Avijit Banerjee, Partha Pratim Mondal and Prof. Gourhari Das constructed a full order observer using the concept of generalized matrix inverse [10]. Alexander Stotsky and Ilya Kolmanovsky presented a paper on unknown input estimation techniques for automotive applications [11]. Unknown input estimation technique for linear discrete-time systems has been given in [7].

However, in this paper the full order observer constructed in [10] has been extended for estimation of unknown input..

II. BRIEF INTRODUCTION TO G-INVERSE

If $A \in \mathbb{R}^{m\times n}$ is a matrix then there exists a unique matrix $A^g \in \mathbb{R}^{n\times m}$, which satisfies the following conditions:

$$(AA^g)^T = AA^g \quad (1)$$
$$(A^gA)^T = A^gA \quad (2)$$
$$AA^gA = A \quad (3)$$
$$A^gAA^g = A^g \quad (4)$$

Consider a system described by linear equation,

$$Ax = y \quad (5)$$

where matrix $A \in \mathbb{R}^{m\times n}$, known vector $y \in \mathbb{R}^m$ and unknown vector $x \in \mathbb{R}^n$. Eq. (5) is consistent if and only if,

$$AA^gy = y \quad (6)$$

Now, if eq. (5) is consistent then the general solution of eq. (5) is given by

$$x = A^gy + (I - A^gA)r \quad (7)$$

([1] Graybill 1969 p. 104). Where $r \in \mathbb{R}^n$ is an arbitrary vector.

III. BRIEF INTRODUCTION TO G-INVERSE

Consider an LTI system described by

$$\dot{x} = Ax + Bu + Ev \quad (8)$$
$$y = Cx \quad (9)$$

where the state vector $x \in \mathbb{R}^n$, known input vector $u \in \mathbb{R}^{m1}$ and the unknown input vector $v \in \mathbb{R}^{m2}$. $y \in \mathbb{R}^m$ is the output vector. A, B, C, and E are known constant matrices with appropriate dimensions. It is assumed that the system is state observable.

The system dynamics in the absence of unknown input can be represented as

$$\dot{x}_u = Ax_u + Bu \quad (10)$$
$$y_u = Cx_u \quad (11)$$

Now, subtracting eq. (10) from eq. (8) and eq. (11) from eq. (9) respectively the state space description of the auxiliary system can be obtained as,

$$\dot{x}_v = Ax_v + Ev \quad (12)$$
$$y_v = Cx_v \quad (13)$$

where $x_v = x - x_u$ is the system response due to unknown input only and $y_v = y - y_u$ is the output response of the system due to unknown input only.

The state of the auxiliary equation can be obtained from eq. (12) and (13). Now the general solution of eq. (13)is

$$x_v = C^g y_v + (I - C^g C)h \quad (14)$$

where $h \in \mathbb{R}^n$ whose elements are arbitrary function of time.

Form eq. (12) and (13) we have

$$\dot{y}_v = CAx_v + CEv \quad (15)$$

From eq. (14) and (15) we have

$$\dot{y}_v = CA(I - C^g C)h + CAC^g y_v + CEv \quad (16)$$

Putting x_v from eq. (14) into eq. (12) we get,

$$(I - C^g C)\dot{h} = A(I - C^g C)h + AC^g y_v + Ev - C^g \dot{y}_v \quad (17)$$

The general solution of eq. (17) for $\dot{h}$ is given by

$$\dot{h} = (I - C^g C)A(I - C^g C)h + (I - C^g C)AC^g y_v + (I - C^g C)Ev + C^g Cz \quad (18)$$

where $z \in \mathbb{R}^n$ is a vector whose elements are arbitrary function of time.

From eq. (18) and (16), the observer dynamic equation can be written as,

$$\dot{\hat{h}} = [(I - C^g C)A(I - C^g C) - KCA(I - C^g C)]\hat{h} + [(I - C^g C)AC^g - KCAC^g]y_v + [(I - C^g C)E - KCE]v + C^g Cz + K\dot{y}_v \quad (19)$$

where K is the observer gain and $\hat{h} \to h$ as $\hat{x} \to x$.

In order to eliminate the first derivative of y_v explicitly present in (15) the following substitution has been made.

$$\hat{h} = \hat{q} + Ky_v \quad (20)$$

and the observer dynamics can be written as

$$\dot{\hat{q}} = [(I - C^g C)A(I - C^g C) - KCA(I - C^g C)]\hat{q} + [(I - C^g C)E - KCE]v + [(I - C^g C)AC^g - KCAC^g + (I - C^g C)A(I - C^g C)K - KCA(I - C^g C)K]y_v + C^g Cp \quad (21)$$

Without loss of generality, the arbitrary vector p may be chosen as a null vector for simplicity and less computation, the observer dynamics can be represented as,

$$\dot{\hat{q}} = [(I - C^g C)A(I - C^g C) - KCA(I - C^g C)]\hat{q} + [(I - C^g C)E - KCE]v + [(I - C^g C)AC^g - KCAC^g + (I - C^g C)A(I - C^g C)K - KCA(I - C^g C)K]y_v \quad (22)$$

A. Condition for existence of UIO

To nullify the effect of the unknown input from observer dynamics the observer gain parameter K can bedesigned such that

$$(I - C^g C)E - KCE = 0 \quad (23)$$

The general solution of K is given by

$$K = (I - C^g C)E(CE)^g + K_v[I - (CE)(CE)^g] \quad (24)$$

where K_v is an arbitrary matrix and the arbitrariness of K is also depends on K_v. Putting the value of K from eq. (24) into eq. (23) we get,

$$(I - C^g C)E(CE)^g(CE) = (I - C^g C)E \quad (25)$$

Eq. (25) is the consistency condition of eq. (23). Putting eq. (24) into eq. (22), it can be shown that the observer matrix would be (A_v - $K_v C_v$) where,

$$A_v = (I - C^g C)A(I - C^g C) - (I - C^g C)E(CE)^g CA(I - C^g C) \quad (26)$$

and $C_v = [I - (CE)(CE)^g]CA(I - C^g C)$ (27)

The arbitrary matrix K_v should be chosen such that the real part of the eigen values of observer matrix $(A_V - K_V C_V)$ become negative.

Then the UIO for auxiliary system, given by eq. (12) and (13) can be obtained as

$$\dot{\hat{q}} = (A_v - K_v C_v)\hat{q} + [(I - C^g C)AC^g - KCAC^g + (I - C^g C)A(I - C^g C)K - KCA(I - C^g C)K]y_v \quad (28)$$

and the estimated state of the auxiliary system can be expressed as

$$\hat{x}_v = (I - C^g C)\hat{q} + [C^g + (I - C^g C)K]y_v \quad (29)$$

The general solution for estimated unknown input can be expressed from eq. (12),

$$\hat{v} = E^g(\dot{\hat{x}}_v - A\hat{x}_v) + (I - E^g E)h_0 \quad (30)$$

where h_0 is any arbitrary vector. From the theory of generalized matrix inverse it can be concluded that if thematrix E is of full rank the unknown input can be estimated as

$$\hat{v} = E^g(\dot{\hat{x}}_v - Ax_v), \text{ since } E^g E = I. \quad (31)$$

IV. NUMERICAL EXAMPLE

For MATLAB simulation, taking the same numerical example of a class of flight pathrate demand missile autopilot as described in [4], in which

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -86 & 0 & -12 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -32400 & -216 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -45360 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, E = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \text{ and } K_v = \begin{bmatrix} -10 & -20 \\ 3 & -4 \\ -7 & 8 \\ -1 & 2 \end{bmatrix}.$$

The system has the initial condition $x_0 = \begin{bmatrix} 20 \\ 0.25 \\ 1.43 \\ 100 \end{bmatrix}$

and the unknown input is taken as $v = 300e^{-0.5t}sin100t$.

Simulation responses for estimation of states and the responses of the system with and without unknown input are given in Fig.4.1-4.4 below, where black doted lines denote the estimated signals, red firm lines indicate the response of the state in presence of known and unknown input both and green firm lines indicate the response of the state without unknown input. The responses for the actual and estimated unknown input are given in Fig. 4.5, where the red firm line denotes the original unknown input signal (v) and the black dotted line denotes the estimated unknown input signal ($\hat{v}$). font.

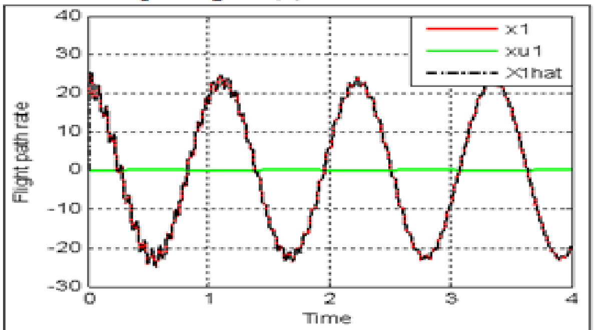

Fig. 4.1: Flight path rate

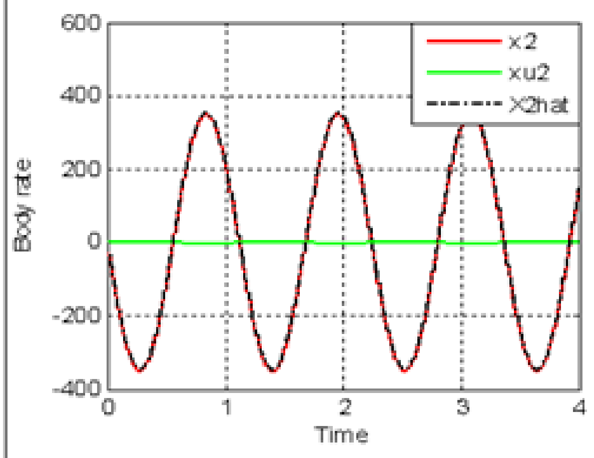

Fig. 4.2: Body Rate

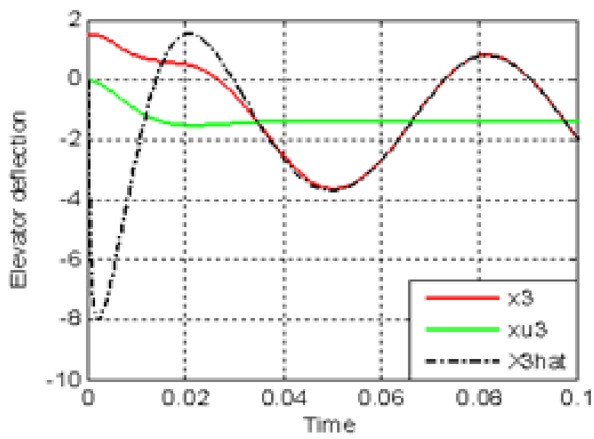

Fig. 4.3: Elevator deflection

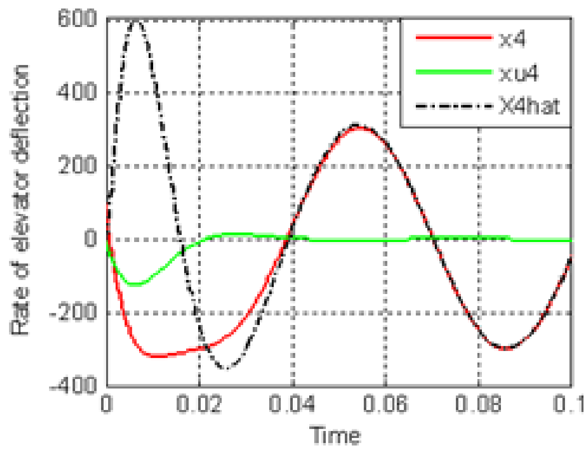

Fig. 4.4: Rate of elevator deflection

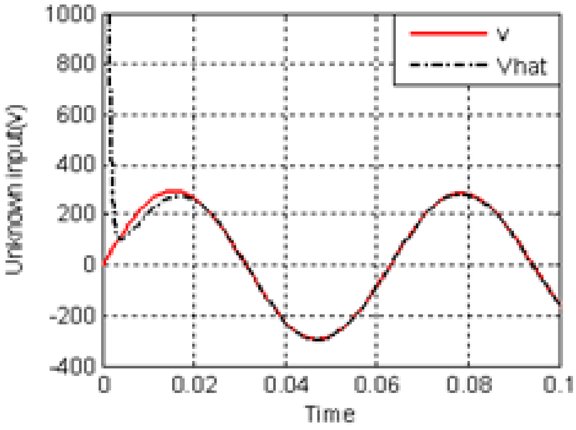

Fig. 4.5: Unknown input (v)

V. CONCLUSIONS

In the proposed method firstly we design an auxiliary system which has the mathematical model in the influence of unknown input (v) only. Now using simple algebraic equation unknown input has been estimated. The necessary conditions are proposed and solved using generalized matrix inverse. Illustrated numerical example with simulated results show the estimated unknown input which tracks the actual unknown input well enough, even if the signal frequency is very high.

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